psyc3010 lecture 11

mixed anova

last week: within-subject anova
next week: a bit on logistic regression, plus
overview of course, T-vals, and ***practice exam***

last week → this week

- last week we returned to anova to consider within-subjects designs
- this week we reunite within-subjects and between-subjects anova – mixed factorial anova: within and between subjects factors
- actually not too difficult because we have kind of done it already
  - every time we do within subjects anova we deal with a between-subjects factor, the random effect of subjects
- **But first, brief Q&A about Assignment 2 ?**

mixed anova

- **also called split-plot anova**
  - Apparently because the first mixed designs emerged in agricultural research where ‘plots’ of land were assigned -> BS treatments as well as divided -> WS factors
  - NB confusion alert:
    - Mixed anova has a BS factor and a WS factor.
    - Mixed model within-subjects ANOVA is the normal way of doing WS ANOVA (where you evaluate sphericity and report an adjusted F, such as GG) – in contrast to MANOVA
- within-subjects anova is great for power, but some variables can be tricky or unethical to manipulate within-subjects
  - e.g., gender, brain injury
- can also manipulate a variable BS to exclude the potential carry-over effects
  - because observations in BS design are independent

assumptions

- **DV is normally distributed**
- **between subjects terms:**
  - homogeneity of variance within levels of between-subjects factor – the ordinary garden-variety homogeneity of variance assumption
- **within-subjects terms:**
  - homogeneity of variance: AxS interactions constant at all levels of B
  - variance-covariance matrix same at all levels of A
  - pooled (or average) variance-covariance matrix exhibits compound symmetry (c.f. sphericity)
  - usual epsilon adjustments apply when within-subjects assumptions are violated

an example

- start with an easy one – just add a between-subjects factor to last week's example
  - we had four blocks of trials in a learning study – each block was a level of the within subjects factor
  - lets say we think a particular bit of the brain is responsible for this particular kind of learning…
  - compare learning of normals (control group) with
    - subjects given brain lesion
    - subjects given drug

(typically this research would use ***rats***)

the split-plot design

time taken (seconds) to complete the maze is the **DV**

group and block are the **IVs**

		BLOCK					
GROUP		1	2	3	4	Totals	
Normal	subject 1	45	42	35	26	148	
	subject 2	35	33	28	15	111	
	subject 3	61	57	48	26	192	
	subject 4	39	36	30	8	113	
Total (normal)		**180**	**168**	**141**	**75**	**564**	
Drug	subject 5	32	30	25	20	107	
	subject 6	48	45	38	37	168	
	subject 7	52	48	40	38	178	
	subject 8	67	63	53	49	232	
Total (drug)		**199**	**186**	**156**	**144**	**685**	
Lesion	subject 9	77	72	60	55	264	
	subject 10	70	66	55	56	247	
	subject 11	70	66	55	60	251	
	subject 12	58	54	45	44	201	
Total (lesion)		**275**	**258**	**215**	**215**	**963**	
block total		654	612	512	434	**2212**	**Grand Total**
block mean		54.50	51.00	42.67	36.17	**46.08**	**Grand Mean**

the split-plot design

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the split-plot design

block is a ***fixed within-subjects factor***

group is a ***fixed between-subjects factor***

GROUP		BLOCK 1	2	3	4	Totals	
Normal	subject 1	45	42	35	26	148	
	subject 2	35	33	28	15	111	
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the split-plot design

because one of the IVs is a within-subjects factor, we include the **random factor *SUBJECTS*** in the partitioning of the variance

		BLOCK 1	2	3	4	Totals	
	subject 1	45	42	35	26	148	
	subject 2	35	33	28	15	111	
	subject 3	61	57	48	26	192	
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the subjects factor is said to be **NESTED** under levels of the **between-subjects factor** GROUP

(Each subject is tested in only 1 group)

the split-plot design

the subjects factor is also said to be **CROSSED** with the **within-subjects** factor BLOCK

each subject participates in each block

	BLOCK 1	2	3	4	Totals	
subject 1	45	42	35	26	148	
subject 2	35	33	28	15	111	
subject 3	61	57	48	26	192	
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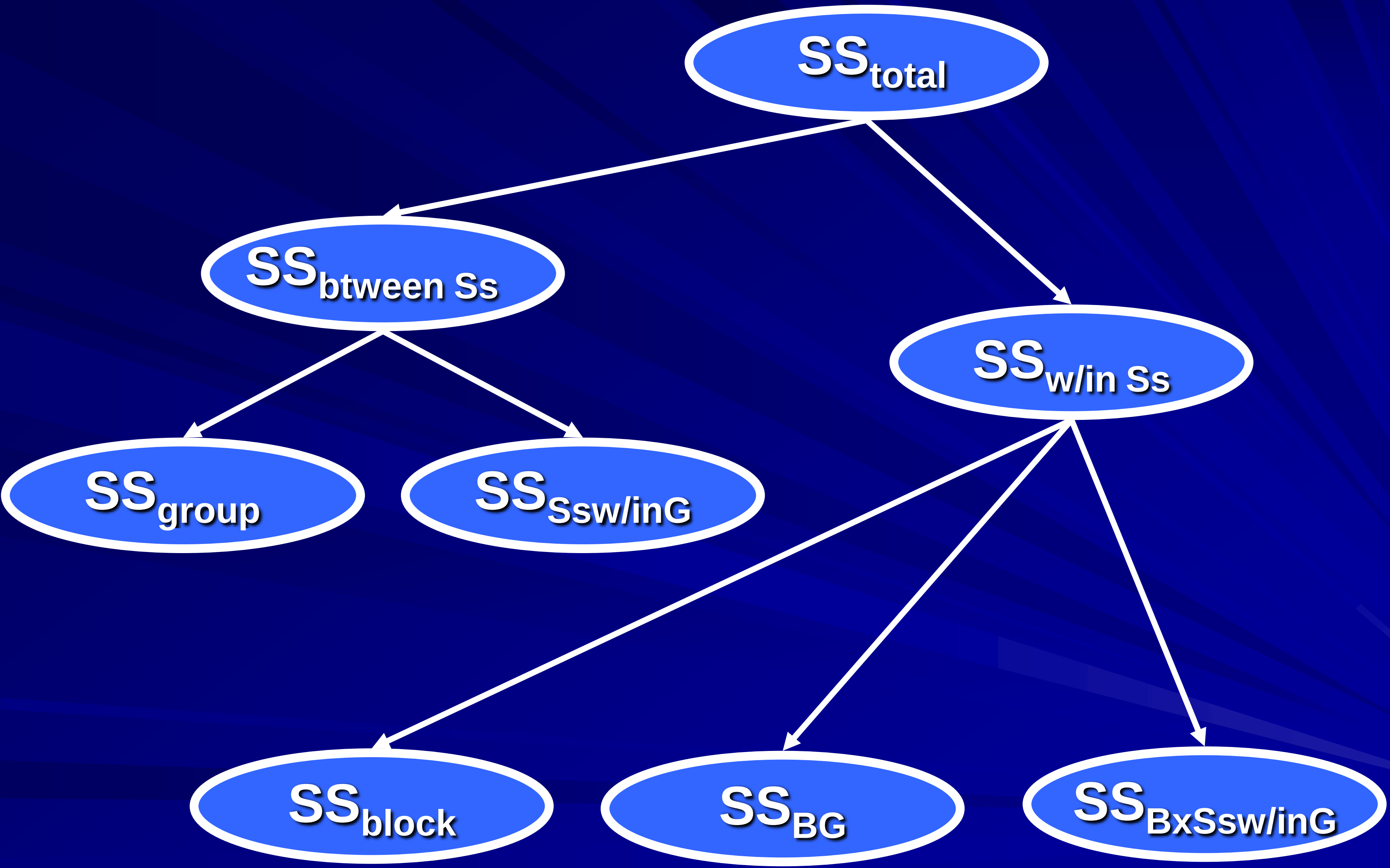
In a two by three mixed ANOVA in which gender (male; female) serves as a between-subjects variable and time of test (start of semester, mid-semester, end of semester) serves as a repeated measures variable, subject is crossed with _______ and nested within ________ .

partitioning the variance:

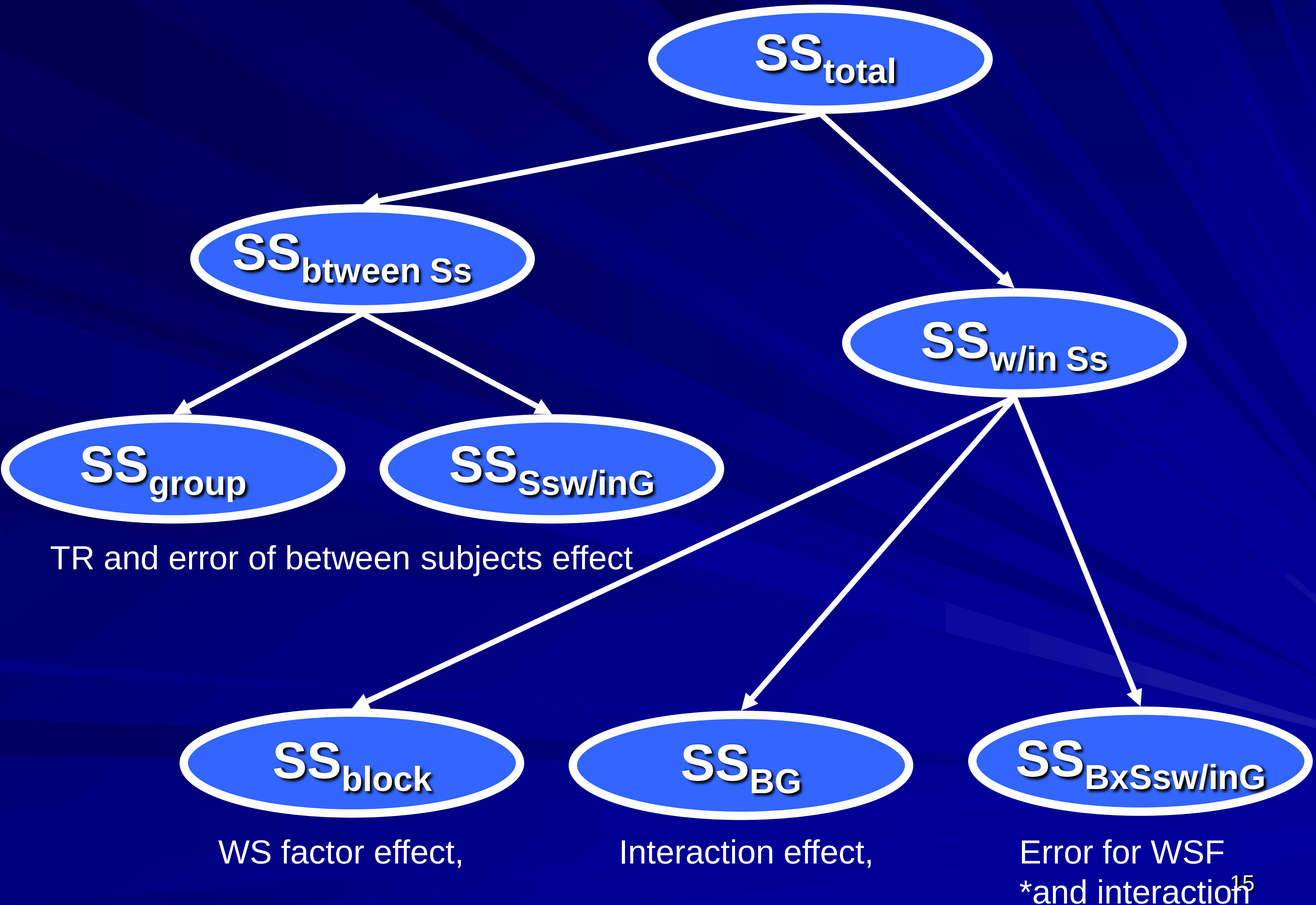


effects and error terms

- **this design is to be treated as a 2-way mixed factorial, so three omnibus effects are to be tested**
  - main effect of group
  - main effect of block
  - group x block interaction

  - ***one error term is required for the between-subjects factor*** (subjects within groups)
  - ***one error term is required for the within-subjects factor and the two-way interaction*** (interaction between block and subjects within groups)
- Point: whereas last week error was TR x S (for fully within factors), for a within factor crossed with a between factor the error for the between ME is Swithingroups and the error for the within ME and the interaction within factor x between factor is $WFxS_{withingroups}$

partitioning the variance:



Understanding the Mixed design

Variability within the groups is error for the BS effect

F = ratio of variability among group means divided by variability within groups

GROUP		BLOCK 1	2	3	4	Totals	
Normal	subject 1	45	42	35	26	148	
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Understanding the Mixed design

Inconsistencies in the Block effect across subjects (SxBlk interaction) are the error for the WS effect

F = ratio of variability among means for WS levels divided by variability in WS effect

		BLOCK					
GROUP		1	2	3	4	Totals	
Normal	subject 1	45	42	35	26	148	
	subject 2	35	33	28	15	111	
	subject 3	61	57	48	26	192	
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Understanding the Mixed design

Inconsistencies in the Block effect across subjects (SxBlk interaction) are the error for the WS x BS interaction (Grp x Block)

F = ratio of variability among cell means for WS levels within each group (adjusted for MEs) divided by variability in WS effect

GROUP		BLOCK 1	2	3	4	Totals	
Normal	subject 1	45	42	35	26	148	
	subject 2	35	33	28	15	111	
	subject 3	61	57	48	26	192	
	subject 4	39	36	30	8	113	
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calculations

$\Sigma X^2 = $ **113832** $\quad \frac{(\Sigma X)^2}{N} = $ **2212^2 / 48 = 101936.33**

$SS_{total} = \Sigma X^2 - \frac{(\Sigma X)^2}{N} = $ **113832 – 101936.33 = 11895.67**

BETWEEN SUBJECTS EFFECTS:

$SS_S = \frac{\sum T_S^2}{b} - \frac{(\Sigma X)^2}{N} = $ **148^2 + ... + 201^2 / 4 – 101936.33 = 8850.17**

$SS_G = \frac{\sum T_G^2}{nb} - \frac{(\Sigma X)^2}{N} = $ **564^2 + 685^2 + 963^2 / 16 – 101936.33 = 5231.79**

$SS_{Ssw/inG} = SS_S - SS_G = $ **8850.17 – 5231.79 = 3618.38**

calculations

WITHIN SUBJECTS EFFECTS:

$SS_{w/inSs} = SStotal - SS_S =$ **11895.67 - 8850.17** $=$ **3045.5**

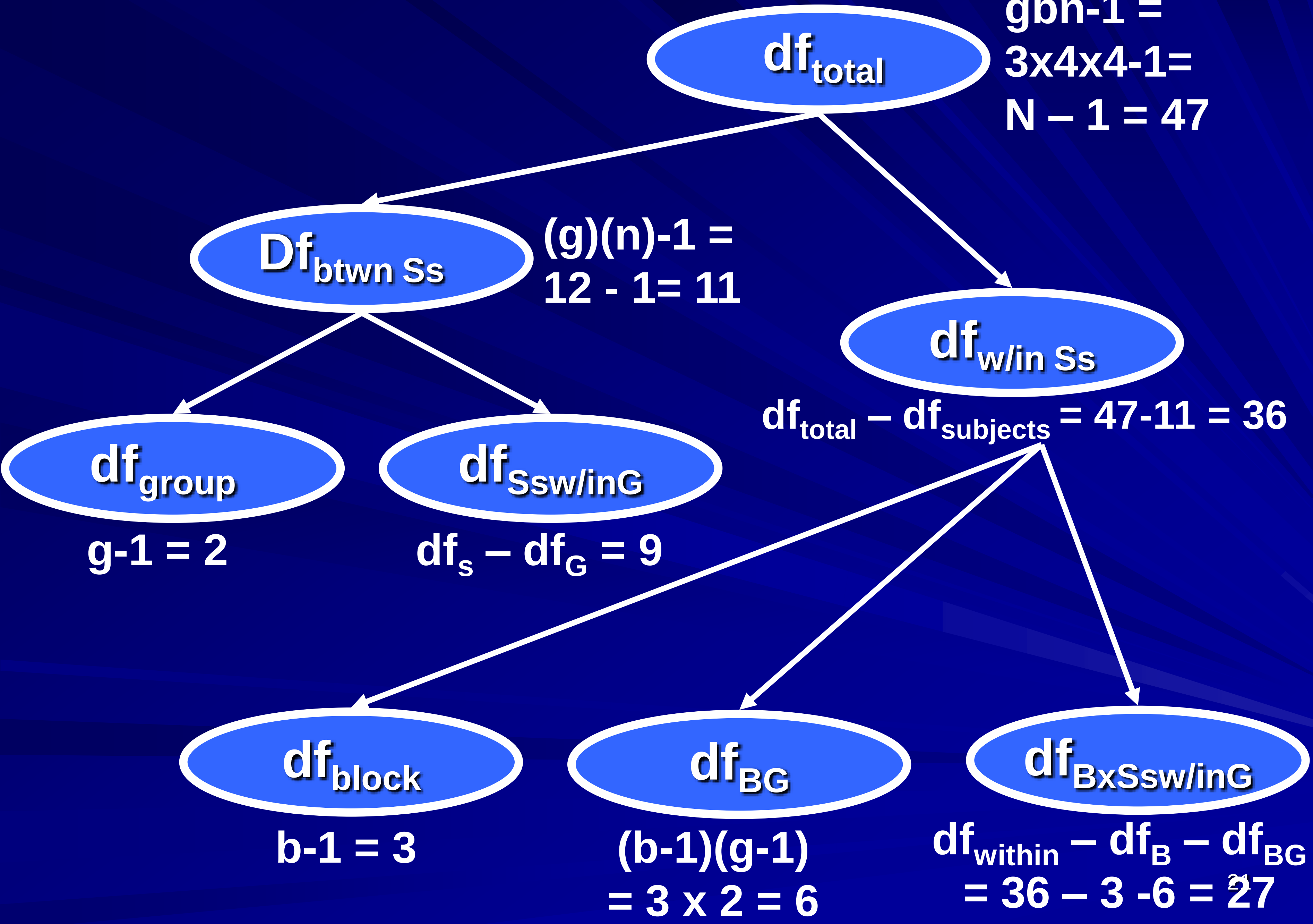
$SS_B = \frac{\sum T_B^2}{ng} - \frac{(\Sigma X)^2}{N}$ $=$ **654^2 + ... + 434^2 / 12 – 101936.33** $=$ **2460.33**

$SS_{cells} = \frac{\sum T_{BG}^2}{n} - \frac{(\Sigma X)^2}{N}$ $=$ **180^2 + ... + 215^2 / 4 – 101936.33** $=$ **8073.17**

$SS_{BG} = SS_{cells} - SS_B - SS_G$ $=$ **8073.17 – 2460.33 – 5231.79 = 381.05**

$SS_{BxS\ w/inG} = SS_{w/inSs} - SS_{BG} - SS_B$ $=$ **3045.5 – 381.05 – 2460.33 = 204.12**

degrees of freedom:



In a two by three mixed ANOVA in which gender (male; female) serves as a between-subjects variable and time of test (start of semester, mid-semester, end of semester) serves as a repeated measures variable, with 20 men and 20 women tested three times each, the degrees of freedom will be:

Df_{total} :______ Df_{WS} :______

Df_{BS} :______ Df_{TIME} :______

Df_{G} :______ Df_{GxTIME} :______

$Df_{SwithinG}$:______ $Df_{TxSwithinG}$:______

summary table

Source	SS	df	MS	F
Between Subjects:	8850.17	11		
Group	5231.79	2	2615.90	6.51 *
Ss w/in Group	3618.38	9	402.04	
Within Subjects	3045.50	36		
Block	2460.33	3	820.11	108.48 *
Block x Group	381.05	6	63.51	8.40 *
Block xSs w/in Group	204.12	27	7.56	
Total	11895.67	47.00		

***Fcrit* (2,9)= 4.26**

***Fcrit* (3,27)= 2.95**

***Fcrit* (6,27)= 2.45**

SPSS output

Tests of Within-Subjects Effects

Measure: MEASURE_1

Source		Type III Sum of Squares	df	Mean Square	F	Sig.
BLOCK	Sphericity Assumed	2460.333	3	820.111	108.478	.000
	Greenhouse-Geisser	2460.333	1.225	2008.476	108.478	.000
	Huynh-Feldt	2460.333	1.633	1506.268	108.478	.000
	Lower-bound	2460.333	1.000	2460.333	108.478	.000
BLOCK * GROUP	Sphericity Assumed	381.042	6	63.507	8.400	.000
	Greenhouse-Geisser	381.042	2.450	155.530	8.400	.005
	Huynh-Feldt	381.042	3.267	116.641	8.400	.001
	Lower-bound	381.042	2.000	190.521	8.400	.009
Error(BLOCK)	Sphericity Assumed	204.125	27	7.560		
	Greenhouse-Geisser	204.125	11.025	18.515		
	Huynh-Feldt	204.125	14.701	13.886		
	Lower-bound	204.125	9.000	22.681		

this error term is Block x Ss w/in Group

SPSS output

Tests of Between-Subjects Effects

Measure: MEASURE_1

Transformed Variable: Average

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Intercept	101936.333	1	101936.333	253.547	.000
GROUP	5231.792	2	2615.896	6.507	.018
Error	3618.375	9	402.042		

this error term is Ss w/in Group

following up main effects

- **between-subjects factor**
  - rule is the same as it would be if this were just a 1-way between-subjects anova:
  - use error term from test of between-subjects main effect
  - $MS_{Ss\ w/in\ G}$ in this case

- **within-subjects factor**
  - use a separate error term (as per last week)
  - $MS_{B_{comp} xS\ w/in\ G}$

significant main effect of group

- ***so if the bit of the brain affected by the lesions and drugs is indeed responsible for the learning in our study, we would expect…***
  – the lesion and drug group to have worse (slower) performance than the normal (control) group
  – the lesion group to perform about the same as the drug group (i.e., same process is being interrupted)
  – could test this with a set of orthogonal linear contrast just like the ones we saw earlier in the semester…

significant main effect of group

mean for G_1 = 35.25

mean for G_2 = 42.81

mean for G_3 = 60.19

calculations for contrast 1

$$SS_{contrast} = \frac{nL^2}{a_j^2}$$

$$L = \sum a_j \overline{X}_j$$

	Group		
	Normal	Drug	Lesion
	35.25	**42.81**	**60.19**
Contrast 1	**2**	**-1**	**-1**
Contrast 2	0	1	-1

L = 2(35.25) – 1(42.81) – 1(60.19) = **-32.5**

$$SS_{contrast} = \frac{(16)(-32.5)^2}{6} = \mathbf{2816.67}$$

calculations for contrast 2

$$SS_{contrast} = \frac{nL^2}{a_j^2}$$

$$L = \sum a_j \overline{X}_j$$

	Group		
	Normal	Drug	Lesion
	35.25	**42.81**	**60.19**
Contrast 1	2	-1	-1
Contrast 2	**0**	**1**	**-1**

$L = 0(35.25) + 1(42.81) - 1(60.19) = \mathbf{-17.38}$

$$SS_{contrast} = \frac{(16)(-17.38)^2}{2} = \mathbf{2416.52}$$

summary table

Source	SS	df	MS	F	
G1 vs G2 & G3	2816.67	1	2816.67	7.01	*
G2 vs G3	2416.52	1	2416.52	6.01	*
Ss w/in Group	3618.38	9	402.04		

$F_{crit}(1,9) = 5.12$

therefore, averaging over the 4 experimental blocks, the normal (control) group performed better than the drug group and lesion group, and the drug group in turn performed better than the lesion group

significant main effect of block

- ***the comparisons between the different groups doesn't really tell us if any <u>learning</u> occurred – we need to see that subjects are completing the maze faster by the end of the study***
  - could test this with a set of linear contrast just like the ones we saw last week…
  - as block is a within-subjects factor we have to get the error term for each comparison based upon only the data involved in that comparison
  - for the sake of brevity – let's just compare the first block with the last

significant main effect of block

as per last week, we get SS_{Bcomp} and the error term by running a 1-way within-subjects anova on our two comparison blocks

SS_{block} is the $SS_{contrast}$ (SS_{Bcomp})

$SS_{blockxS}$ is the error term ($SS_{BcompXS}$)

	BLOCK 1	2	3	4	Totals	
subject 1	45			26	71	
subject 2	35			15	50	
subject 3	61			26	87	
subject 4	39			8	47	
	180			**75**	**255**	
subject 5	32			20	52	
subject 6	48			37	85	
subject 7	52			38	90	
subject 8	67			49	116	
	199			**144**	**343**	
subject 9	77			55	132	
subject 10	70			56	126	
subject 11	70			60	130	
subject 12	58			44	102	
	275			**215**	**490**	
block total	654			434	**1088**	**Grand Total**
block mean	54.50			36.17	**45.33**	**Grand Mean**

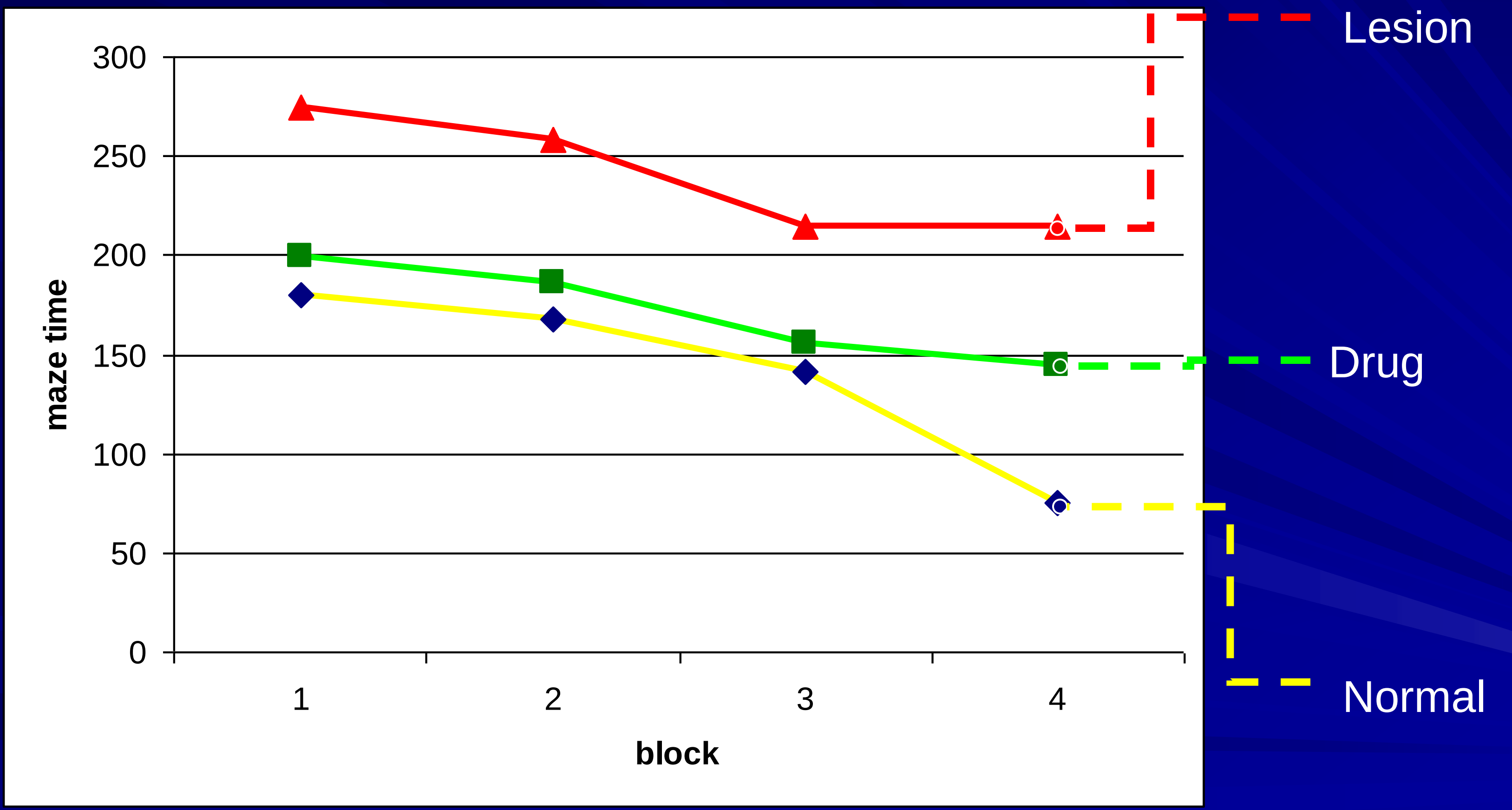
summary table

Source	SS	df	MS	F
Block 1 vs Block4	2016.67	1	2016.67	65.76 *
Error	337.33	11	30.67	

$F_{crit}(1, 11) = 4.84$

therefore, averaging over the 3 experimental groups, subjects were performing significantly better by the end of the experiment – hence learning has occurred

interaction of group x block



simple effects
within-subjects factors…

- if we wanted to conduct the simple effects of ***block*** (for each group), we just run a 1-way within subjects anova on block separately for each group
  - as such, the error term used will be appropriate for each effect
  - using the pooled error term (in this case, $MS_{BxSsw/inG}$) is not appropriate, and may over or underestimate error component (denominator of F-ratio), even when degrees of freedom are adjusted

summary table

Source	SS	df	MS	F	
Normal					
Block	1651.50	3	550.50	38.71	*
Error	128.00	9	14.22		
Drug					
Block	490.69	3	163.56	58.74	*
Error	25.06	9	2.78		
Lesion					
Block	699.19	3	233.06	41.08	*
Error	51.06	9	5.67		

***Fcrit(3,9)* = 3.86**

note that the *average* of these error terms = (14.22 + 2.78 + 5.67) / 3 = 7.56: the value of our $MS_{B\ xSs\ w|in\ G}$

simple effects
between-subjects factors…

could also examine simple effects of group for each of the four blocks – here we have two possible approaches:

- use a separate error term for each simple effect
- i.e., run four 1-way between-subjects anovas to compare groups at each of the four blocks
- *then use* $\boldsymbol{MS_{Ss\ w/in\ G\ at\ B1}}$ *,* $\boldsymbol{MS_{Ss\ w/in\ G\ at\ B2}}$ ***etc***

OR . . . .

- a special pooled error term may be used: $\boldsymbol{MS_{Ss\ w/in\ cell}}$
- this error term is an estimate of the average error variance within the 12 cells
  - $SS_{w/in\ cell} = SS_{Sw/inG} + SS_{BxSs\ w/in\ G}$
  - $MS_{Ss\ w/in\ cell} = SS_{w/in\ cell} / (df_{Ss\ w/in\ G} + df_{BxSs\ w/in\ G})$

    (is OK to pool because between subjects effects should be independent)

simple effects
between-subjects factors…

- in both cases, the sums of squares for the simple effects are derived just as we have seen in the case of between subjects anova (see lecture 3)
- the separate error term method is a little quicker, but you compromise on degrees of freedom

summary table
separate error term

Source	SS	df	MS	F	
Group at Block1	1263.50	2	631.75	4.74	*
Ss w/in G at B1	1199.50	9	133.28		
Group at Block2	1134.00	2	567.00	4.81	*
Ss w/in G at B2	1062.00	9	118.00		
Group at Block3	765.17	2	382.58	4.56	*
Ss w/in G at B3	755.50	9	83.94		
Group at Block4	2450.17	2	1225.08	13.69	*
Ss w/in G at B4	805.50	9	89.50		

***Fcrit* (2,9) = 4.26**

NB

df are just from the block main effect

(aside: might be informative to calculate estimates of effect size – the bigger effect is clearly occurring at block 4)

summary table pooled error term ($MS_{Ss\ w/in\ cell}$)

Source	SS	df	MS	F	
Group at Block1	1263.50	2	631.75	5.95	*
Ss w/in G at B1	3822.50	36	106.18		
Group at Block2	1134.00	2	567.00	5.34	*
Ss w/in G at B2	3822.50	36	106.18		
Group at Block3	765.17	2	382.58	3.60	*
Ss w/in G at B3	3822.50	36	106.18		
Group at Block4	2450.17	2	1225.08	11.54	*
Ss w/in G at B4	3822.50	36	106.18		

***Fcrit* (2,36) = 2.94**

NB

$SS_{Ss\ w|in\ cell}$ = 204.12 + 3618.38 (SSw/inGrp + SSgrpxblkxS)

summary table
pooled error term ($MS_{Ss\ w/in\ cell}$)

Source	SS	df	MS	F	
Group at Block1	1263.50	2	631.75	5.95	*
Ss w/in G at B1	3822.50	36	106.18		
Group at Block2	1134.00	2	567.00	5.34	*
Ss w/in G at B2	3822.50	36	106.18		
Group at Block3	765.17	2	382.58	3.60	*
Ss w/in G at B3	3822.50	36	106.18		
Group at Block4	2450.17	2	1225.08	11.54	*
Ss w/in G at B4	3822.50	36	106.18		

***Fcrit* (2,36) = 2.94**

NB

df = 9 + 27

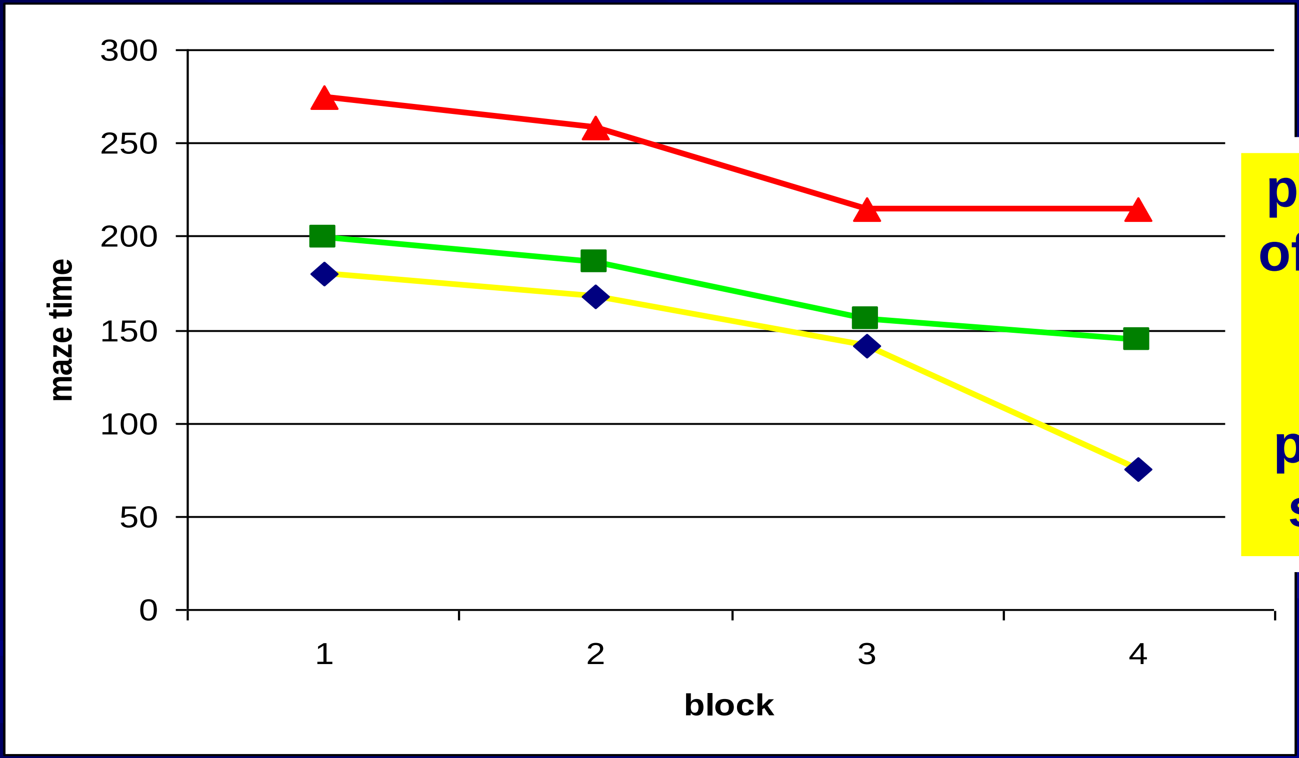
simple simple effects…

- we could conduct follow-up comparisons for the 3 groups at block 4 – this would be identical to the follow up for the main effect of group we did earlier – I.e. could use linear contrasts with MSSwithinG@B4 [same error term as original simple effect]
- but for now, I think we have all had enough!

summary of findings for the study…

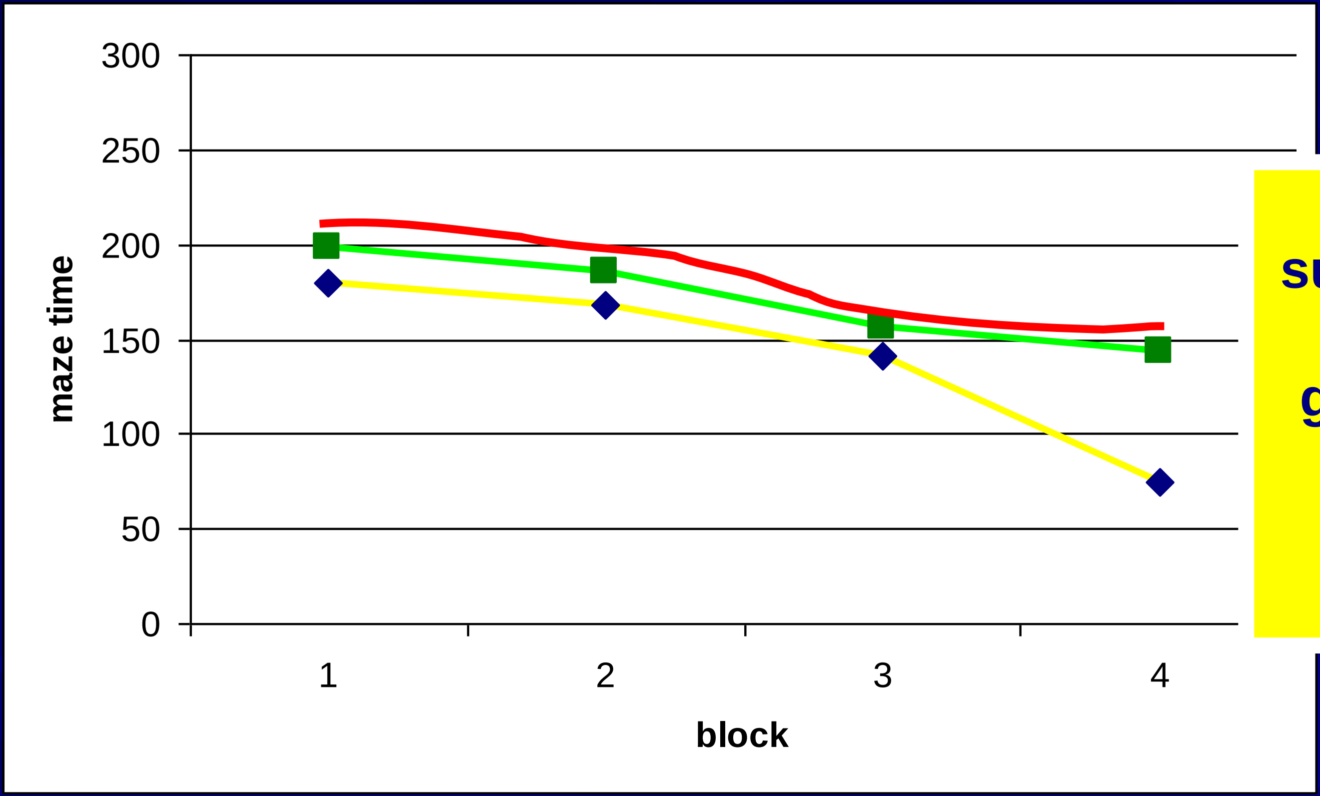
- significant effect of block
  – faster times at block 4 than block 1 (indicates learning)
- significant effect of group
  – normal (control) group was faster than the drug group which was faster than the lesion group
- significant interaction
  – learning was occurring for each group
  – groups were performing at a different level at each block
  – if we followed this up further we might find that the largest differences were in block 4

mixed support overall…



possibly some kind of confound leading to the lesion group's performance being severely impaired

mixed support overall…



Damage was supposed to impair <u>learning</u>, so this graph would have been a more theoretically pleasing result

bottom line

- the really tricky stuff with within-subjects and mixed anova is sorting out the error terms
- the logic is similar though:
  - The error term for between main effects is subjects within groups
  - The appropriate error term for within effects is always the effect being examined in interactions with the random factor subjects

In class next week:

- Brief bit on logistic regression
- Overview of course themes as I see them; general pontificating
- T-VALS
- Discussion of exam and practice exam

In the tutes:

- This week: Consult for A2
- Next week: Practice exam

readings :

- Howell Ch. 14 pp. 577-582
- Logistic Regression in Field: Ch 16-16.5,16.6