## psyc3010 lecture 5

## consolidation of between-subjects anova power

last week: complex anova next week: Blocking and ANCOVA

## last week $\rightarrow$ this week

- last week we looked at how a 2-way anova can be extended to a 3-way anova, where three factors (IVs) are included in the model
- this week we discuss the assignment
- briefly recap on 3-way anova
- Including simple interactions, simple simple effects, and simple simple comparisons
- A quick note on unequal $n$
- Then on to power
- What it is, graphically \& statistically
- effect size measures, d and $\Phi^{\prime}$ (phi prime)
- non-centrality parameters $\delta$ (delta) and $\Phi$ (phi)
- What influences power


# last week's experiment with some different data... 

| Males |  |  |  | Females |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Reinforcement |  |  |  | Reinforcement |  |  |
| Personality | Rew | None | Pun | Personality | Rew | None | Pun |
| Impulsive | 350 | 425 | 450 | Impulsive | 350 | 355 | 460 |
|  | 345 | 430 | 465 |  | 320 | 350 | 495 |
|  | 330 | 445 | 465 |  | 330 | 340 | 485 |
| Total | 1025 | 1300 | 1380 | Total | 1000 | 1045 | 1440 |
| Mean | 341.67 | 433.33 | 460.00 | Mean | 333.33 | 348.33 | 480.00 |
| Anxious | 485 | 450 | 470 | Anxious | 320 | 360 | 480 |
|  | 450 | 455 | 480 |  | 340 | 360 | 470 |
|  | 495 | 445 | 465 |  | 350 | 370 | 450 |
| Total | 1430 | 1350 | 1415 | Total | 1010 | 1090 | 1400 |
| Mean | 476.67 | 450.00 | 471.67 | Mean | 336.67 | 363.33 | 466.67 |

## omnibus tests in a 3-way anova

- main effects:
- differences between marginal means of one factor (averaging over levels of other factors)
- two-way interactions (first-order):
- examines whether the effect of one factor is the same at every level of a second factor (averaging over a third factor)
- three-way interaction (second-order):
- examines whether the two-way interaction between two factors is the same at every level of the third factor
- or whether the cell means differ more than you would expect given the main effects and the two-way interactions


## main summary table (from sPPs)

Tests of Between-Subjects Effects
Dependent Variable: RT

| Source | Ty pe III Sum <br> of Squares | df | Mean Square | F | Sig. |
| :--- | ---: | ---: | ---: | ---: | ---: |
| REINFORC | 60237.500 | 2 | 30118.750 | 169.418 | .000 |
| PERSONAL | 7802.778 | 1 | 7802.778 | 43.891 | .000 |
| GENDER | 24544.444 | 1 | 24544.444 | 138.062 | .000 |
| REINFORC * PERSONAL | 7751.389 | 2 | 3875.694 | 21.801 | .000 |
| REINFORC * GENDER | 16426.389 | 2 | 8213.194 | 46.199 | .000 |
| PERSONAL * GENDER | 6944.444 | 1 | 6944.444 | 39.062 | .000 |
| REINFORC * PERSONAL | 6601.389 | 2 | 3300.694 | 18.566 | .000 |
| *GENDER | 4266.667 | 24 | 177.778 |  |  |
| Error | 134575.000 | 35 |  |  |  |
| Total |  |  |  |  |  |


is interaction significant?
(repeat this sequence for AC at B 1 and B 2 , and BC at A1 and A2, to explore interaction

- or pick based on theory / hypotheses)
is $A \times B$ at
C1 or C2
significant?


## calculate simple interaction effects


are the simple simple effects of $A$ or B significant?

conduct tests for simple simple comparisons

## following up the 3-way interaction

- simple interaction effects
- tests of 2-way interactions at each level of the third factor
- e.g., personality x reinforcement at each level of gender
- simple simple effects:
- tests of the effect of a factor at each level of second factor within levels of a third factor
- e.g., effects of reinforcement at each level of personality (for males)
- simple simple comparisons:
- compares effect of different levels of a factor at a particular level of a second factor within levels of a third factor
- e.g., effects of rew versus pun or neutral for impulsive males


## summary table for follow-up tests

Simple Interaction Effects

| PR at G1 | 13744.44 | 2 | 6872.22 | 38.656 | 0.000 |
| :--- | :---: | :---: | ---: | :---: | :---: |
| PR at G2 | 608.33 | 2 | 304.17 | 1.711 | 0.202 |
| Error | 4266.67 | 24 | 177.78 |  |  |

Simple Simple effects in the interaction of PR at G1

| R at P1 at G1 | 23116.67 | 2 | 11558.33 | 65.016 | 0.000 |
| :--- | :---: | ---: | ---: | ---: | ---: |
| R at P2 at G1 | 538.89 | 2 | 269.44 | 1.516 | 0.240 |
| Error | 4266.67 | 24 | 177.78 |  |  |

Simple Simple comparisons in simple effect of R at P1 at G1

| R vs $\mathbf{N}$ and P at P1 at G1 | 22047.90 | 1 | 22047.90 | 124.019 | 0.000 |
| :--- | :---: | :---: | ---: | :---: | :---: |
| N vs P at P1 at G1 | 355.64 | 1 | 355.64 | 2.000 | 0.170 |
| Error | 4266.67 | 24 | 177.78 |  |  |

## summary table for follow-up tests

Simple Interaction Effects

PR at G1
PR at G2
Error
13744.44
608.33
4266.67
2

2
24
6872.22
38.656
0.000
304.17
1.711
0.202
the personality $x$ reinforcement interaction is...

- significant for males
- non-significant for females
we can follow up this significant simple interaction effect of Personality x Reinforcement for males using simple simple effects


## summary table for follow-up tests

the effect of reinforcement (for males) is

- significant for the impulsive group
- not-significant for the anxious group

Simple Simple effects in the interaction of PR at G1

| R at P1 at G1 | 23116.67 | 2 | 11558.33 | 65.016 | 0.000 |
| :--- | :---: | :---: | ---: | :---: | :---: |
| R at P2 at G1 | 538.89 | 2 | 269.44 | 1.516 | 0.240 |
| Error | 4266.67 | 24 | 177.78 |  |  |

we can follow up the significant simple simple effect of reinforcement at impulsivity (for males) using simple simple comparisons

## summary table for follow-up tests

simple simple comparisons...

- calculated using the t-test formula (c.f. lecture 3)
- can also be calculated using a F-test
(last week's tutorial exercise showed how to do this)

Simple Simple comparisons in simple effect of R at P1 at G1

R vs $\mathbf{N}$ and $\mathbf{P}$ at $\mathbf{P 1}$ at G1 22047.90
$\mathbf{N}$ vs $\mathbf{P}$ at P 1 at $\mathbf{G 1}$
Error
4266.67

24
22047.90
124.019
355.64
2.000
177.78
0.000
0.170

## summary table for follow-up tests

for impulsive males,

1) performance under rewarding feedback was significantly different from that under neutral or punishing feedback
2) performance under punishing feedback was not significantly different to performance under neutral feedback

Simple Simple comparisons in simple effect of R at P1 at G1

| R vs $\mathbf{N}$ and P at P1 at G1 | 22047.90 | 1 | 22047.9 | 124.019 | 0.000 |
| :--- | :---: | :---: | ---: | :---: | :---: |
| N vs P at P1 at G1 | 355.64 | 1 | 355.644 | 2.000 | 0.170 |
| Error | 4266.67 | 24 | 177.78 |  |  |

## reporting

1. I haven't put effect sizes. These would be required for all omnibus tests. Effect sizes are usually included for simple effects as well, and even for simple comparisons.

A $2 \times 2 \times 3$ factorial ANOVA was performed, and a significant three-way interaction was observed among personality, gender, and reinforcement $F(2,24)=169.42, p<.001$. The simple personality x reinforcement was significant for men, $F(2,24)=$ $38.66, p<.001$, but not for women, $F(2,24)=1.71, p=.202$. Simple simple effects of reinforcement were significant for impulsive males, $F(2,24)=65.02, p<.001$, but not for anxious males, $F(2,24)=1.52, p=.240$. Planned contrasts showed that impulsive males performed better with reward ( $M=341.67$ ) than with punishment or no reinforcement ( $M=441.67$ ), $F(1,24)=$ 124.02, $p<.001$, while the performance of impulsive males receiving no reinforcement ( $M=433.33$ ) was not significantly different to those receiving punishment ( $M=460.00$ ), $F(2,24)=$ $2.00, p=.170$.

## reporting

2. NB we are reporting simple simple effects of reinforcement now - assumes this = theoretical focus. Last week we hypothesized different personality effects depending on reinforcement, but this week we are focusing on different reinforcement effects depending on personality.
Results of a $2 \times 2 \times 3$ factorial ANOVA indica ed a significant threeway interaction among personality, gender, and reinforcement $F(2,24)=169.42, p<.001$. The simple perso ality $x$ reinforcement interaction was significant for males, $F(2,24)=38.66, p<.001$, but not for females, $F(2,24)=1.71, p=.202$. There was a significant simple simple effect of reinforcement for impulsive males, $F(2,24)=65.02$, $p<.001$, but not for anxious males, $F(2,24)=1.52, p=.240$. Planned contrasts showed that impulsive males performed better with reward ( $M=341.67$ ) than with punishment or no reinforcement ( $M=$ 441.67), $F(1,24)=124.02, p<.001$, while the performance of impulsive males receiving no reinforcement ( $M=433.33$ ) was not significantly different to those receiving punishment ( $M=460.00$ ), $F(2,24)=2.00, p=.170$.

## reporting

> 3. NB for simplicity we reported for women that the simple interaction of $\mathrm{P} \times \mathrm{R}$ was ns, but unless interested ONLY in interaction we would also 'followup' ns simple interaction by looking at simple effect of Reinforcement for women (collapsing across Personality), if hypotheses suggest $R=$ the theoretically key IV. We want to know if there are effects of reinforcement for women that don't change depending on personality.

Results of a $2 \times 2 \times 3$ factorial ANOVA indicated a/significant three-way interaction among personality, gender, and reinforcement $F(2,24)=$ 169.42, $p<.001$. The simple personality $x$ reiporcement interaction was significant for males, $F(2,24)=38.66, p<.01$ A, but not for females, $F(2,24)=1.71, p=.202$. [Insert before simple simple effects of $R$ for men:] For women, across personality type the simple effect of reinforcement was / was not significant, $F(2,24)=X$, $p=$, eta2=.... [lf yes, would also follow-up with contrasts.] In addition, there was a significant simple simple effect of reinforcement for impulsive males, $F(2,24)=$ $65.02, p<.001$, but not for anxious males, $F(2,24)=1.52, p=.240$. etc"

## Reporting

- Where there is one key IV with two moderators of interest, you might also choose to skip directly from a three-way to the simple simple effects.
- E.g. (made-up data), "The three-way interaction was significant, $F=, p=$, eta2=. Follow-up tests examined the simple simple effects of reinforcement within each level of personality and gender. Differences in reaction to reinforcement were observed for anxious men, $F=, \mathrm{p}=$,eta2=, and impulsive women, $\mathrm{F}=, \mathrm{p}=$, eta2=. Specifically, [describe simple simple comparisons here]. For impulsive men and anxious women, no effects of reinforcement were found, Fs< \#\#, ps> \#\#, eta2s< \#\#."
- In fact you might even go directly from the threeway to the simple comparisons (and put these in a table) to reduce your wordiness and the complexity of the text.

Table 1. Mean reaction time as a function of gender, personality, and reinforcement.

1. Most experimental journals would also want to see standard deviations.

> Women

Reinforcement:

| Impulsive | $485.33_{\mathrm{a}}$ | $481.67_{\mathrm{a}}$ | $320.00_{\mathrm{b}}$ |
| :--- | :--- | :--- | :--- |
| Anxious | $487.10_{\mathrm{a}}$ | $485.33_{\mathrm{a}}$ | $323.00_{\mathrm{b}}$ |
| Impulsive | $484.255_{\mathrm{a}}$ | $478.76_{\mathrm{a}}$ | $316.67_{\mathrm{b}}$ |
| Anxious | $367.55_{\mathrm{a}}$ | $455.67_{\mathrm{b}}$ | $583.00_{\mathrm{c}}$ |

Note. Differing subscripts within the same row indicate significant differences among the means.
E.g. (made-up data): "The three-way interaction of $P \times R \times G$ was significant, $\mathrm{F}=, \mathrm{p}=$,eta2=. Table 1 displays the means by gender, personality, and reinforcement to decompose the interaction. As seen in Table 1, follow-up analyses revealed that for women and for impulsive men, reinforcement led to faster responses (more learning) than either the control condition or punishment, which did not differ. For anxious men, by contrast, punishment produced the most learning, followed by the control condition, with reinforcement being the least successful condition (slowest response times)."

## Table 1. Mean reaction time as a

 function of gender, personality, and reinforcement.2. NB how this hides all stats (except the threeway) - excellent for nonstats minded reviewers

Reinforcement:
Women
Impulsive
Anxious
Men
Impulsive
Anxious

Punishment Neutral 481.67 a $320.00_{\mathrm{b}}$ $485.33_{\mathrm{a}} \quad 323.00_{\mathrm{b}}$ $+78.76_{a}$ $455.67_{b}$
316.67b $583.00_{c}$

Note. Differing subscripts within the same row indicate significant differences among the means.
E.g. (made-up data): "The three-way interaction of $P \times R \times G$ was significant, $\mathrm{F}=, \mathrm{p}=$, eta2=. Table 1 displays the means by gender, personality, and reinforcement to decompose the interaction. As seen in Table 1, follow-up analyses revealed that for women and for impulsive men, reinforcement led to faster responses (more learning) than either the control condition or punishment, which did not differ. For impulsive men, by contrast, punishment produced the most learning, followed by the control condition, with reinforcement being the least successful condition (slowest response timess)."

## Reporting: The bottom line

- With higher-order designs, there's a lot more flexibility about how you structure the analyses / write-up
- It's best to be guided by your theory.
- This may require you to fully explore the simple interactions, simple simple effects, simple simple comparisons, etc. within the text...
- Or allow for some skipping.
> The more complex the design, the more likely that you need a Table (or Tables, and/or figures) to report the results.


## What happens with Unequal n?

- Something really complex mathematically - so you're not required to learn this material (it will not be assessed on the exam)
- We provide a quick overview of the issues for the interested among you on the Blackboard web site in the practice materials section
- Take home message: If unequal $n$ are too extreme (ratio of largest to smallest cell size >3:1), results of analyses are unstable (less likely to replicate - can have more Type 1 and more Type 2 errors).
- Always try for equal n in study designs where anticipate using ANOVA.
- Avoid using natural group variables like gender or ethnicity in ANOVA if ratio of people in groups exceeds $3: 1$ for largest: smallest.


## power - introduction

- See Howell sec. 11.13, 13.7
- significant differences are defined with reference to a criterion, or threshold (controlled/acceptable rate) for committing type-1 errors, typically . 05 or . 01
- the type-1 error $\rightarrow$ finding a significant difference in the sample when it actually doesn't exist in the population
- type-1 error rate denoted $\alpha$
- in the development of hypothesis testing procedures, little attention has been paid to the type-2 error
- the type-2 error $\rightarrow$ finding no significant difference in the sample when there is a difference in the population
- type-2 error rate denoted $\beta$


## power

shifts our focus to the type-2 error rate:
official definition:"the probability of correctly rejecting a false $H_{0}$ " - mathematically works out to 1- $\beta$
useful definition: the degree to which we can detect treatment effects
(incl main effects, interactions, simple simple comparisons etc)

## Reality vs Statistical Decisions

 Reality:| Statistical Decision: |
| :--- | |  | $H 1$ |
| :---: | :---: |
| Reject H0 | "False alarm" |
| $\alpha$ | "Hit" <br> (grooviness!) <br> $1-\beta$ <br> Power |
| Fail to reject H0 | "Hit" <br> (grooviness!) <br> $1-\alpha$ |
|  | "Miss" <br> $\beta$ |

## power

## Common constraints :

-Cell size too small

- B/c sample difficult to recruit or too little time / money
-Small effects are often a focus of theoretical interest (especially in social / clinical / org everything "real world")
- DV is subject to multiple influences, so each IV has small impact
- "Error" or residual variance is large, because many IVs unmeasured in experiment / survey are influencing DV
- Interactions are of interest, and interactions draw on smaller cell sizes (and thus lower power) than tests of main effects [Cell means for interaction are based on $n$ observations, while main effects are based on $n \times$ \# of levels of other factors collapsed across]


## power - kinds of questions

1) "I have done a study and I want to report the power of my significant effect"

- need to calculate observed power (post hoc power)

2) "I have done a study and did not find a significant effect: how could I increase power to detect the difference?"

## or

3) "I am designing a study and want to know how many subjects I need in order to detect my predicted effect"

- need to calculate predicted power (a priori power)


## in terms of hypothesis testing



## factors affecting power

- power depends on a number of things
- significance level, $\alpha$
- relaxed $\alpha \rightarrow$ more power
- sample size, $\mathbf{N}$
- more $\mathrm{N} \rightarrow$ more power
- mean differences, $\mu_{0}-\mu_{1}$
- larger differences $\rightarrow$ more power
- error variance $-\sigma_{e}{ }^{2}$ or MS $_{\text {error }}$
- less error variance $\rightarrow$ more power
power as a function of $\alpha$



## power as a function of $\alpha$



## power as a function of $\mu_{0}-\mu_{1}$



## power as a function of $\mu_{0}-\mu_{1}$



## as a function of $n$ and $\sigma^{2}$



## as a function of n and $\sigma^{2}$



## as a function of effect size (d)

- power closely related to effect size
- effect size is determined by some of these previous factors we looked at ( $\mu_{0}-\mu_{1}$ and $\sigma$ ):
- $\mathrm{d}=\frac{\mu_{1}-\mu_{0}}{\sigma}$
 $\frac{\text { how much the means differ }}{\text { variability in the sample }}$
- d indicates about how many standard deviations the means are apart, \& thus the overlap of the two distributions:

| effect size | d | percentage overlap |
| :--- | :---: | :---: |
| Small | 0.20 | $85 \%$ |
| Medium | 0.50 | $67 \%$ |
| Large | 0.80 | $53 \%$ |

## put them all together...

- thus, we can calculate power if we know the effect size and the sample size
- for a t-test, this information can be combined using $\delta$, a non-centrality parameter:


## $\delta=\mathrm{d} \sqrt{\mathrm{n}}$

- if we already have our data we know $d$ and $n$
- can then calculate power by computing $\delta$ and looking up tables to see what power we have (post hoc power)
- if we don't have our data, we can specify $\delta$ and d
- can then work out the $\boldsymbol{n}$ we need to get a certain level of power (a priori power)


## Power \& Effect

- Smaller effect sizes harder to find significant difference - need more powerful test
- "Magnifying glass"
- Big effect needs a less powerful test (magnifying glass)
- Small effect needs a more powerful test


## ...extend to 1-way anova...

- for anova the notation changes slightly:

$$
\delta=\mathrm{d} \sqrt{\mathrm{n}} \text { becomes } \phi=\phi^{\prime} \sqrt{\mathrm{n}}
$$

- where $\mathrm{n}=$ number of observations in each treatment group
- $\Phi^{\prime}=$ effect size (with 2 groups is same as d)
- $\Phi^{\prime}$ calculation is conceptually similar to d:

where $\mu=$ grand mean
$\mu_{j}=$ each group mean
$\mathrm{k}=$ number of groups
$\boldsymbol{\sigma}_{\boldsymbol{e}}{ }^{2}=$ pooled variance (use $\mathrm{MS}_{\text {error }}$ )


## power for 1-way anova

- lets go back to our 'limericks’ study...
- suppose we didn't include the distraction factor - our data would look like this:

this is the error term we would have got running the study as a 1-way anova


## power for effect of consumption

$$
\phi^{\prime}=\sqrt{\frac{\sum \tau_{\mathrm{j}}^{2}}{\mathrm{k} \sigma_{\mathrm{e}}^{2}}}=\sqrt{\frac{\sum\left(\mu_{\mathrm{j}}-\mu\right)^{2}}{\mathrm{k} \sigma_{\mathrm{e}}^{2}}}
$$

where $\mu=$ grand mean $=58.33$

$$
\begin{aligned}
& \mu_{j}=\text { each group mean }=63.75,64.69,46.56 \\
& \mathrm{k}=\text { number of groups }=3 \\
& \sigma_{e}{ }^{2}=\text { pooled variance }\left(\text { use } \mathrm{MS}_{\text {error }}\right)=125.21
\end{aligned}
$$

$$
\begin{aligned}
& \phi^{\prime}=\sqrt{\frac{(63.75-58.33)^{2}+(64.69-58.33)^{2}+(46.56-58.33)^{2}}{3 \times 125.21}} \\
& =0.56
\end{aligned}
$$

$\phi=\phi^{\prime} \sqrt{\mathrm{n}}=0.56 \sqrt{16}=2.22$

## power for effect of consumption

- look up Noncentral $F(n c F)$ tables in Howell,

$$
\text { for } \begin{aligned}
\Phi & =2.22 \text { with: } \\
& -\mathrm{df}_{\text {treat }} \text { of } 2(\mathrm{k}-1=3-1) \\
& -\mathrm{df}_{\text {error }} \text { of } 45(\mathrm{~N}-\mathrm{k}=48-3) \\
& -\alpha=.05
\end{aligned}
$$

- this tells us the value of $\beta$, in this case 0.09
- as power = $1-\beta$, we have .91 power
- that is, we have an $91 \%$ chance of detecting our effect (and correctly rejecting the null hypothesis), if the effect is real in the population
- Cohen (1992) suggests $>.80$ is optimal


## how about 2-way factorial anova?

formula is basically the same, only that now we have 3 effects to estimate the power for:

$$
\begin{aligned}
& \phi \alpha^{\prime}=\sqrt{\frac{\sum \alpha_{\mathrm{j}}^{2}}{\mathrm{a} \sigma_{\mathrm{e}}^{2}}}=\sqrt{\frac{\sum\left(\mu_{\mathrm{j}}-\mu\right)^{2}}{\mathrm{a} \sigma_{\mathrm{e}}^{2}}} \quad \phi \alpha=\phi \alpha^{\prime} \sqrt{\mathrm{nb}} \\
& \phi \beta^{\prime}=\sqrt{\frac{\sum \beta_{\mathrm{k}}^{2}}{\mathrm{~b} \sigma_{\mathrm{e}}^{2}}}=\sqrt{\frac{\sum\left(\mu_{\mathrm{k}}-\mu\right)^{2}}{\mathrm{~b} \sigma_{\mathrm{e}}^{2}}} \phi \beta=\phi \beta^{\prime} \sqrt{\mathrm{na}}
\end{aligned}
$$

$$
\phi \alpha \beta=\sqrt{\frac{\sum \alpha \beta_{\mathrm{jk}}^{2}}{\mathrm{ab} \sigma_{\mathrm{e}}^{2}}}=\sqrt{\frac{\sum\left(\mu-\mu_{\mathrm{j}}-\mu_{\mathrm{k}}+\mu_{\mathrm{jk}}\right)^{2}}{\mathrm{ab} \sigma_{\mathrm{c}}^{2}}}
$$

$$
\phi \alpha \beta=\phi \alpha \beta \sqrt{n}
$$

## power for 2-way anova

- back to our 'limericks' study...
- this time, we include the distraction factor:

|  | mean creativity rating |  | (marginal mean) |
| :---: | :---: | :---: | :---: |
|  | Distracted | Controls |  |
| 0 pints of beer | 66.88 | 60.63 | 63.75 |
| 2 pints of beer | 66.88 | 62.50 | 64.69 |
| 4 pints of beer | 35.63 | 57.50 | 46.56 |
| (marginal mean) | 56.46 | 60.21 |  |
| $\mathrm{MS}_{\text {error }}=83.02$ | $\mu=58.33$ |  | $N=48$ |

## power for main effect of consumption

$$
\phi \alpha^{\prime}=\sqrt{\frac{\sum \alpha_{i}^{2}}{\mathrm{a} \sigma_{\mathrm{c}}^{2}}}=\sqrt{\frac{\sum\left(\mu_{\mathrm{j}}-\mu\right)^{2}}{\mathrm{a} \sigma_{\mathrm{c}}^{2}}}
$$

$$
\begin{aligned}
& \phi \alpha^{\prime}=\sqrt{\frac{(63.75-58.33)^{2}+(64.69-58.33)^{2}+(46.56-58.33)^{2}}{3 \times 83.02}} \\
& =.915
\end{aligned}
$$

$$
\begin{aligned}
& \phi \alpha=\phi \alpha^{\prime} \sqrt{n b} \\
& =0.915 \sqrt{8 x 2} \\
& =3.66
\end{aligned}
$$

$$
\left(\mathrm{df}_{\text {treat }}=2(3-1) ; \mathrm{df}_{\text {error }}=42(48-6) ; \alpha=.05\right)
$$

$$
\text { this time } \beta=0.00 \text { (say } 0.001 \text { ) so power }=.99
$$

thus we have a 99\% chance of detecting a significant effect if one exists in the pop.
(and correctly rejecting the null hypothesis)

## what if we had fewer subjects?

- our main effect of consumption is so large that we would have enough power to detect it with much fewer subjects
- we could estimate what the power would be if we had done the study again with 3 people per cell ( $\mathrm{N}=18$ ):

$$
\begin{aligned}
& \begin{array}{l}
\phi=\phi^{\prime} \sqrt{n b}=0.915 \times \sqrt{3 \times 2}=2.24 \\
\mathrm{df}_{\text {treat }} \text { of } 2(=3-1) \\
\quad-\mathrm{df}_{\text {error }} \text { of } 12(18-6) \\
-\alpha=.05
\end{array}
\end{aligned}
$$

- this time $\beta=.14$ so we have .86 power
- that is, we have an $86 \%$ chance of detecting our effect (and correctly rejecting the null hypothesis, if effect exists in the pop)


## power for main effect of distraction

$$
\phi \beta^{\prime}=\sqrt{\frac{\sum \beta_{\mathrm{k}}^{2}}{\mathrm{~b} \sigma_{\mathrm{e}}^{2}}}=\sqrt{\frac{\sum\left(\mu_{\mathrm{k}}-\mu\right)^{2}}{\mathrm{~b} \sigma_{\mathrm{e}}^{2}}}
$$

$$
\phi \beta^{\prime}=\sqrt{\frac{(56.46-58.33)^{2}+(60.21-58.33)^{2}}{2 \times 83.02}}=0.206
$$

$$
\begin{aligned}
& \phi \beta=\phi \beta^{\prime} \sqrt{n a} \\
& =0.206 \sqrt{8 x 3} \\
& =1.01
\end{aligned}
$$

$$
\left(\mathrm{df}_{\text {treat }}=1(2-1) ; \mathrm{df}_{\text {error }}=42(48-6) ; \alpha=.05\right)
$$

this time $\beta=.71$ so we have .29 power that is, we have an 29\% chance of detecting our effect (and correctly rejecting the null hypothesis, if a real effect exists in the pop)

## what if we had more subjects?

-our main effect of distraction is so small that we would need to have a lot more power to detect it
-we could estimate what the power would be if we had done the study again with 30 people per cell $(\mathrm{N}=360)$ :

$$
\begin{aligned}
& \phi \beta=\phi \beta^{\prime} \sqrt{n a} \\
& =0.206 \sqrt{30 x 3} \\
& =1.95
\end{aligned}
$$

$$
\mathrm{df}_{\text {treat }}=1(2-1) ; \mathrm{df}_{\text {error }}=354(360-6) ; \alpha=.05
$$

this time $\beta=.19$ so we have .81 power
that is, we have an $81 \%$ chance of detecting our effect (and correctly rejecting the null hypothesis, if real diff exists in the pop) ${ }_{49}$

## What about higher-order ANOVA?

- Can generalize from this formula for each effect
- E.g. in 3-way ANOVA, look at power for each of the 7 effects (main effects of $A, B$, and $C$; $A B, B C$, and $A C$ interactions; and ABC interaction)


## determining sample size

- if we can estimate our effect size and error we can plan studies in terms of how many participants we need to get .80 power
- often use previous studies to derive your estimates for MSE and effect size
- substituting different values of $n$ into the formula and looking up the ncF tables is one (tedious) way of doing things...


## G*POWER

- G*POWER is a FREE program that can make the calculations a lot easier
http://www.psycho.uni-duesseldorf.de/abteilungen/aap/gpower3/
G*Power computes:
- power values for given sample sizes, effect sizes, and alpha levels (post hoc power analyses),
- sample sizes for given effect sizes, alpha levels, and power values (a priori power analyses)
- suitable for most major statistical methods
- V useful for hons thesis writing or report writing.


## side issues...

- recall the logic of calculating estimates of effect size (i.e., criticisms of significance testing)
- the tradition of significance testing is based upon an arbitrary rule leading to a yes/no decision
- power illustrates further some of the caveats with significance testing
- with a high $N$ you will have enough power to detect a very small effect
- if you cannot keep error variance low a large effect may still be non-significant


## side issues...

- on the other hand...
- sometimes very small effects are important
- by employing strategies to increase power you have a better chance at detecting these small effects


## how can we maximize power?

- power can be increased a number of different ways (some of these we have seen already...)
- increase sample size
- increase a level (usually not wise!)
- decrease error variance
- improving data collection methods:
- procedural
- psychometric
- improving methods of analysis
-i.e., use more powerful statistics


## some methods we will look at...

 methodological:- blocking (next week)
statistical:
- analysis of covariance, ancova (next week)
- within-subjects anova (week 11 \& 12)


## Next week in class:

- Blocking and ANCOVA


## Readings for this week:

- Howell
- Chapter 16 (section 16.7)
- Review chapter 9
- Field
- Chapter 9

In the tutes:

- This week: SPSS tute
- Next week: Assignment 1 consult.

