

# psyc3010 lecture 4

## **interpretation of anova higher order designs (complex anova)**

Before Ekka: following up effects & magnitude of effects in 2-way  
next week: power analysis

# last week → this week

- Before the break, we looked at how to follow-up significant main effects and interactions, and how to calculate effect size
- this week we briefly consider interpretation of factorial anova, before moving on to higher order factorial designs (sometimes called 'complex anova')
- We also distribute Assignment 1 (which can also be downloaded from the course website)

# topics for this week

- **interpreting 2-way factorial ANOVA**
  - ☞ review of omnibus tests + follow-up effects
  - ☞ notes on reporting effects
- **introduction to higher-order designs**
- **omnibus tests in 3-way factorial ANOVA**
  - ☞ main effects
  - ☞ 2-way interactions
  - ☞ 3-way interactions
- **following up 3-way factorial ANOVA**
  - ☞ simple interaction effects
  - ☞ simple simple effects
  - ☞ simple simple comparisons

# wrapping up the distraction study:

## hypotheses we might have had for our study...

1) we predict that creativity will be higher when more alcohol is consumed

*(hence, we predict a **main effect** of consumption)*

2) we predict that creativity will be lower when distracted

*(hence, we predict a **main effect** of distraction)*

3) we predict that the effect of consumption on creativity ratings will be stronger for distracted participants

*(hence, we predict **an interaction** between distraction and consumption)*

# interpretation

a main effect of  
***consumption***

consumption has  
> 2 levels (0, 2 or  
4 pints) so we  
need to conduct  
**follow-up tests**  
to interpret

Summary Table – from lectures 2 and 3

Source	df	SS	MS	F	sig
C (cons)	2	3332.3	1666.15	20.07	.000
D (dist)	1	168.75	168.75	2.03	.161
C x D	2	1978.12	989.06	11.91	.000
Error	42	3487.5	83.02		
Total	47	8966.7			

i.e., there is a significant difference among the ***marginal means*** for consumption ...

“Here’s a set of Linear Contrasts I prepared earlier...”

these are the marginal means for consumption from our data table earlier

a set of weights ( $a_j$ ) is used to define the contrasts:  
contrast 1 compares 0 vs 2 & 4  
contrast 2 compares 2 vs 4

	Consumption		
	0 pints	2 pints	4 pints
	63.75	64.69	46.56
Contrast 1	2	-1	-1
Contrast 2	0	1	-1

$$t = \frac{L}{\sqrt{\frac{\sum a_j^2 MS_{error}}{n * d}}}$$

$$L = \sum a_j \bar{X}_j$$

$$df_{error} = N - ab$$

results of linear contrasts:  
comparison 1:  $t'(42) = 2.91, p < .05$   
comparison 2:  $t'(42) = 5.63, p < .05$

$t'_{\alpha=.05} (42) = 2.33$  (Bonferroni adj)

# interpretation

a main effect of  
*distraction*

main effect is not  
significant so no  
further analysis is  
needed

Summary Table – from lectures 2 and 3

Source	df	SS	MS	F	sig
C (cons)	2	3332.3	1666.15	20.07	.000
D (distr)	1	168.75	168.75	2.03	.161
C x D	2	1978.12	989.06	11.91	.000
Error	42	3487.5	83.02		
Total	47	8966.7			

i.e., there is no significant difference among the *marginal means* for distraction



# interpretation

an interaction between  
*consumption and distraction*

a significant  
interactions needs  
to be followed up  
with **simple  
effects**

Summary Table – from lectures 2 and 3

Source	df	SS	MS	F	sig
C (cons)	2	3332.3	1666.15	20.07	.000
D (distr)	1	168.75	168.75	2.03	.161
<b>C x D</b>	<b>2</b>	<b>1978.12</b>	<b>989.06</b>	<b>11.91</b>	<b>.000</b>
Error	42	3487.5	83.02		
Total	47	8966.7			

i.e., the **cell means** differ, such that differences among the means  
for consumption are not the same for distracted and non-  
distracted



# a note on interactions...

- some sources suggest that once you find a significant interaction you should ignore the main effects as the main effects have been “*qualified*” by the interaction
- this is because the interaction *may* require you to change the interpretation given by the main effect alone (but then again, it may *not* – see Howell section 13.3)
- **ultimately, there is no simple rule:** what you report depends **entirely** upon your research predictions – if you predict a main effect, then report that main effect (and any follow-up tests)
  - in our case, we made a specific prediction about both of our main effects, so we should deal with them accordingly

# reporting

I haven't put effect sizes in for the follow-up comparisons / contrasts; most do nowadays esp. if report Fs.

“Results indicated a significant main effect of consumption,  $F(2,42) = 20.07$ ,  $p < .001$ ,  $\omega^2 = .34$ . Linear contrasts with a Bonferroni adjustment for 2 comparisons indicated that creativity ratings were significantly lower after 2 or 4 pints than after consuming no alcohol,  $t'(42) = 2.91$ ,  $p < .05$  ( $Ms = 63.75, 55.63$ ), and were lower after 4 pints than after 2 pints,  $t'(42) = 5.63$ ,  $p < .05$  ( $Ms = 64.69, 46.56$ ). There was no significant main effect for distraction, indicating that creativity ratings for distracted participants' limericks ( $M = 56.46$ ) were not significantly different from those for controls ( $M = 60.21$ ),  $F(1,42) = 2.03$ ,  $p = .16$ ,  $\omega^2 = .01$ . There was, however, a significant interaction between consumption and distraction, indicating that the effect of consumption was different for distracted and control participants,  $F(1,42) = 11.91$ ,  $p < .001$ ,  $\omega^2 = .20$ .”

NB Interaction needs following up in results section (simple effects + simple comparisons if nec.).

Discuss: although the predicted main effect of alcohol consumption was significant, the direction of the effect was contrary to hypotheses: alcohol lowered creativity ratings. Also the predicted effect of distraction was not significant.

## Following up the significant interaction - Simple Effects from last week:

2 simple effects are significant

Source	SS	df	MS	F	p
<b>C at D1</b>	<b>5208.33</b>	<b>2</b>	<b>2604.17</b>	<b>31.36</b>	<b>0.000</b>
C at D2	102.08	2	51.04	0.61	0.546
D at C1	156.25	1	156.25	1.88	0.177
D at C2	76.56	1	76.56	0.92	0.342
<b>D at C3</b>	<b>1914.06</b>	<b>1</b>	<b>1914.06</b>	<b>23.05</b>	<b>0.000</b>
Error	3487.5	42	83.04		

F at alpha=.05 (2,42) = 3.23

F at alpha=.05 (1,42) = 4.08

if obtained F exceeds critical F **reject the null hypothesis**

**There is a significant effect of distraction at the third level of consumption: the mean creativity ratings for distracted and control participants who have consumed 4 pints are significantly different (no follow up needed as only 2 levels)**

# Following up the significant interaction - Simple Effects from last week:

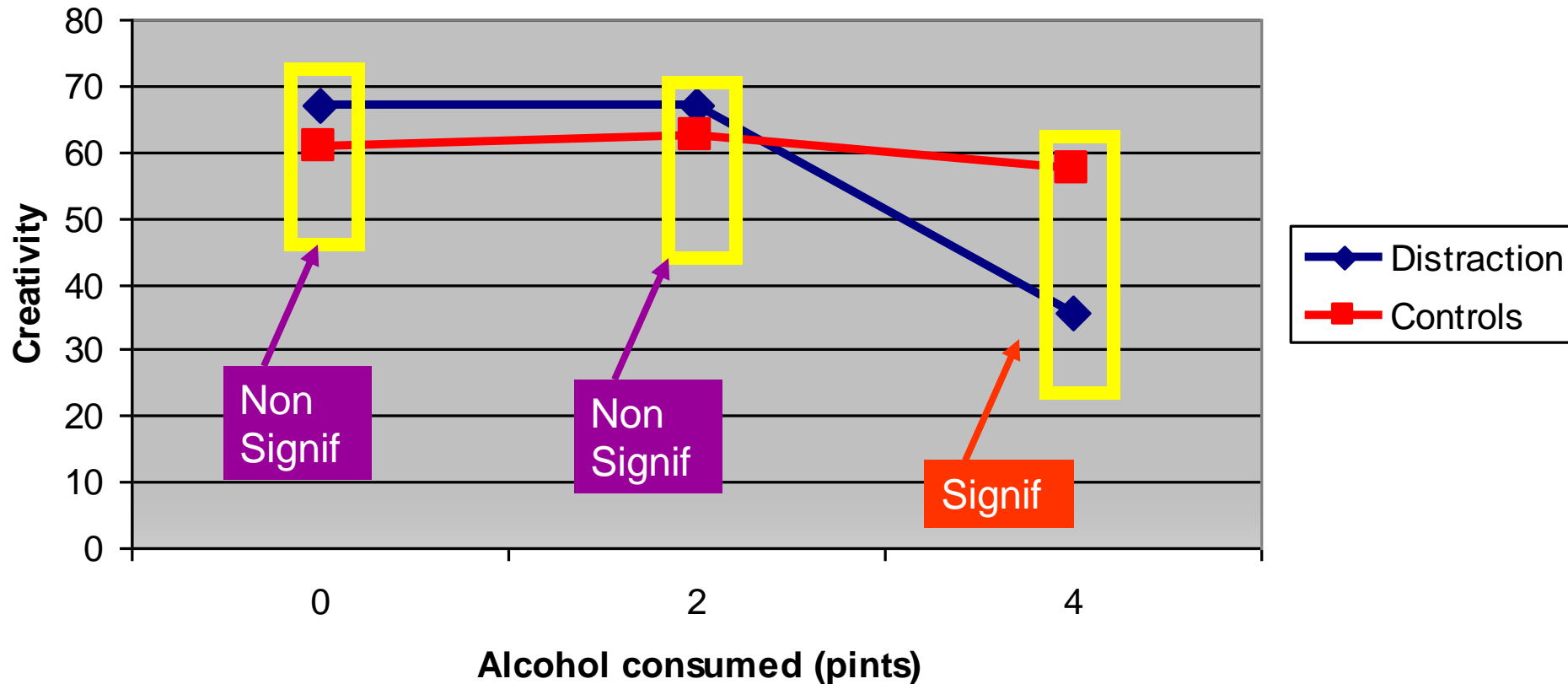
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<b>C at D1</b>	<b>5208.33</b>	<b>2</b>	<b>2604.17</b>	<b>31.36</b>	<b>0.000</b>
C at D2	102.08	2	51.04	0.61	0.546
D at C1	156.25	1	156.25	1.88	0.177
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<b>D at C3</b>	<b>1914.06</b>	<b>1</b>	<b>1914.06</b>	<b>23.05</b>	<b>0.000</b>
Error	3487.5	42	83.04		

F at alpha=.05 (2,42) = 3.23      if obtained F exceeds critical F **reject the null hypothesis**  
 F at alpha=.05 (1,42) = 4.08

There is a significant effect of consumption at the first level of distraction: the mean creativity ratings for distracted differ depending upon whether they have had 0, 2 or 4 pints (follow-up tests needed to identify *where* the difference is)

the ***simple effects of distraction*** describe the differences in creativity between distracted and controls ***at each level of alcohol consumed***



# Following up the significant interaction - Simple Effects from last week:

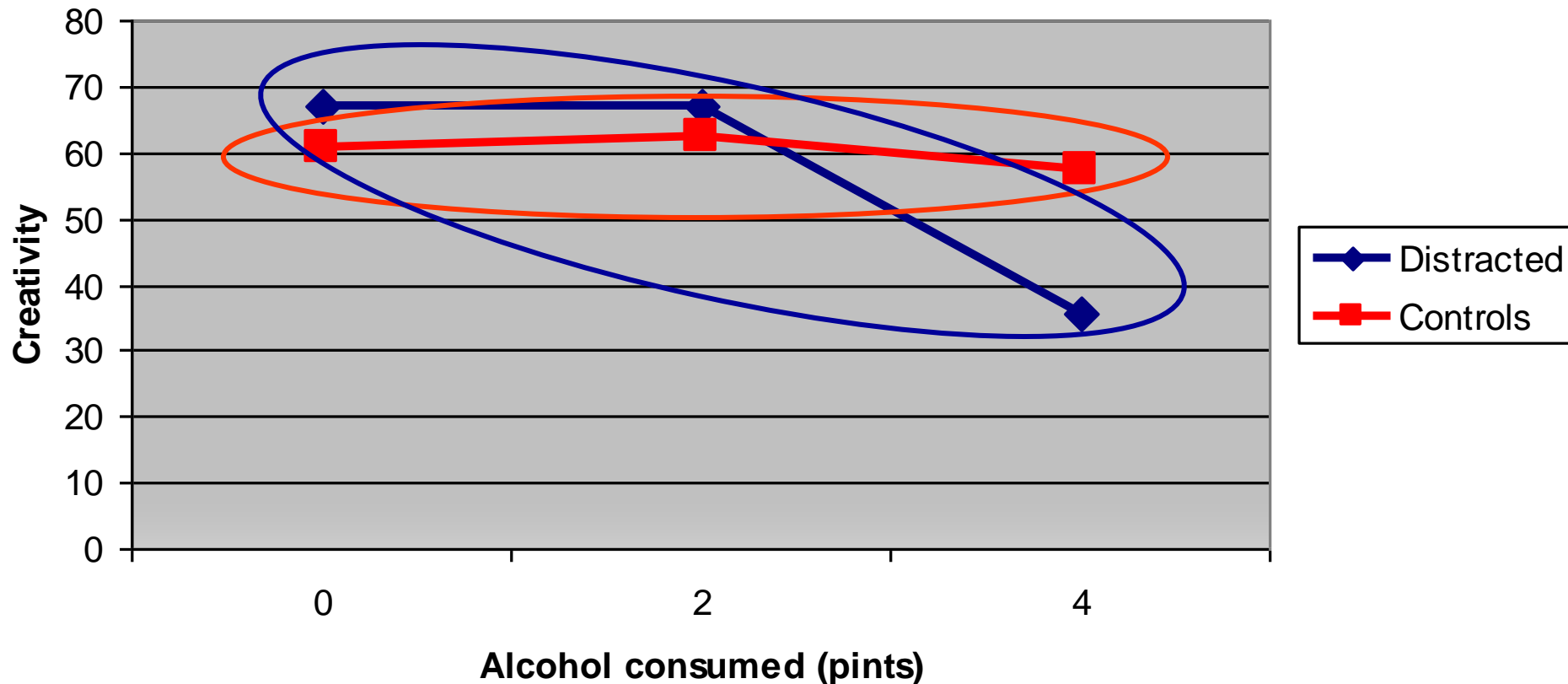
2 simple effects are significant

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Error	3487.5	42	83.04		

F at alpha=.05 (2,42) = 3.23      if obtained F exceeds critical F **reject the null hypothesis**  
 F at alpha=.05 (1,42) = 4.08

**There is a significant effect of consumption at the first level of distraction: the mean creativity ratings for distracted differ depending upon whether they have had 0, 2 or 4 pints**

the ***simple effects of alcohol consumed*** describe the differences in creativity after 0, 2 or 4 pints consumed at ***each level of distraction***



(follow-up tests needed to identify *where* the difference is)



# Following up the significant Simple Effects of consumption for distracted – simple comparisons from last week:

	Consumption		
	0 pints	2 pints	4 pints
<b>Distracted</b>	66.88	66.88	35.63
Contrast 1	2	-1	-1
Contrast 2	0	1	-1

$$t = \frac{L}{\sqrt{\frac{\sum a_j^2 MS_{error}}{n}}}$$

$$L = \sum a_j \bar{X}_j$$

$$df_{error} = N - ab$$

results of linear contrasts:

comparison 1:  $t(42) = 3.96, p < .05$

comparison 2:  $t(42) = 6.86, p < .05$

$t_{\alpha=.05} (42) = 2.33$  (Bonferroni adj)

# a note on simple effects...

- it is preferable to not report all sets of simple effects, for 2 reasons:

- a) the more simple effects we calculate, the greater our risk of making a type 1 error (see Howell, p.436)
- b) usually both sets of simple effects will communicate *similar* information - redundancy
  - so, in our case we would want to report *either* the simple effects of distraction (at each level of consumption) *or* the simple effects of consumption (at each level of distraction)

- **ultimately, there is no simple rule:** what you report depends **entirely** upon your research predictions.

- in our case, we specifically predicted that “the effect of consumption on creativity ratings will be stronger for distracted participants than for controls”. Therefore, we would want to report the simple effects for **consumption** (and associated simple comparisons / contrasts)

# reporting

I haven't put effect sizes in for the follow-up comparisons / contrasts ; most do now.... Effect sizes for simple effects are also required. NB if you calc  $\omega^2$  for controls it works out to -.01 – a meaningless % (% cannot be negative), so set to zero. Another reason some prefer to report  $\eta^2$ .

“...To follow up the significant two-way interaction, the simple effects of consumption were analysed at each level of distraction. There was a significant simple effect of consumption for distracted participants,  $F(2,42) = 31.36, p < .001, \omega^2 = .56$ , but not for controls,  $F(2,42) = 0.61, p = .546, \omega^2 = .00$ . The significant effect of consumption for distracted participants was followed up with Linear contrasts using a Bonferroni adjustment for 2 comparisons. These indicated that, for distracted participants, creativity ratings were lower after 2 or 4 pints than after consuming no alcohol,  $t'(42) = 4.52, p < .001$  ( $M_s = 66.88, 51.26$ ), and also lower after 4 pints than after 2 pints,  $t'(42) = 6.86, p < .001$  ( $M_s = 66.88, 35.63$ ).”

Discuss: the hypothesis was confirmed that the effect of consumption on creativity will be stronger for distracted participants than for controls.



# higher-order factorial designs

- **inclusion of more than 2 independent variables (factors)**
  - Three, four, five .... The world is complex
- **Consideration of more interactions**
  - In Gender (Male/Female) x Age (Young/Old) x Nationality (Australian / American) design
    - Does gender interact with age? Does gender interact with ethnicity? Does age interact with ethnicity?
    - *Three* two-way interactions considered!
  - *And* the exciting possibility that there is **a three-way interaction** between age, gender and nationality!

# Higher order designs

- E.g., 2 (age) x 3 (alcohol) x 2 (sex) between subjects design = 12 cells

Men

Women

No alc 1 drink 5 drinks

No alc 1 drink 5 drinks

Old



Young



# Higher order designs

- **main effects for each IV:**
  - differences between marginal means of the factor (averaging over other factors)
- **two-way interactions:**
  - examines whether the effect of one factor is the same at every level of another factor (averaging over the third factor)
- ***three-way interaction:***
  - examines whether the two-way interaction between two factors is the same at every level of the third factor
  - Or: is there variability in the cell means which is not accounted for by the main effects of the IVs and the 2-way interactions?



# partitioning the variance

## main effects

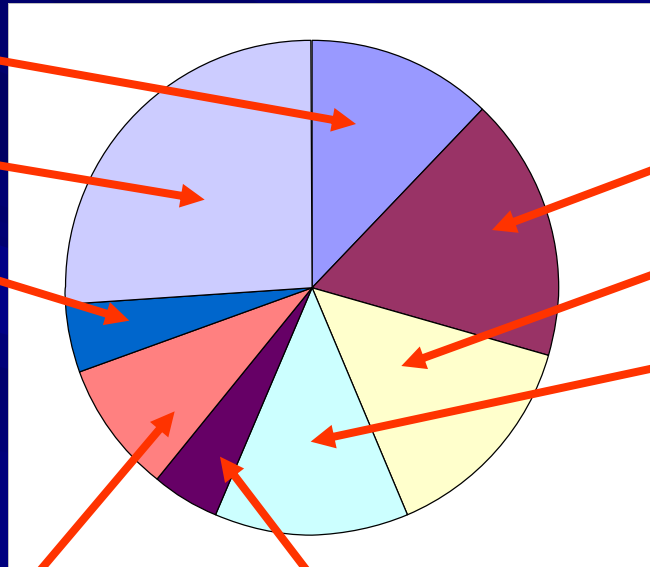
variance due to  $\alpha$

variance due to  $\beta$

variance due to  $\gamma$

## 3-way interaction

variance due to  $\alpha\beta\gamma$



## 2-way interactions

variance due to  $\alpha\beta$

variance due to  $\beta\gamma$

variance due to  $\alpha\gamma$

## error/residual

variance due to  $e$

# higher-order factorial designs

- **inclusion of more than 2 independent variables (factors)**
- linear model for a 2-way factorial design:

$$X_{ijk} = \mu + \alpha_j + \beta_k + \alpha\beta_{jk} + e_{ijk}$$

- linear model for a 3-way design

$$X_{ijkl} = \mu + \alpha_j + \beta_k + \gamma_l + \alpha\beta_{jk} + \beta\gamma_{kl} + \alpha\gamma_{jl} + \alpha\beta\gamma_{jkl} + e_{ijkl}$$

# 3-way data table

- Age (2) x alcohol (3) x sex (2)

Men

No alc 1 drink 5 drinks

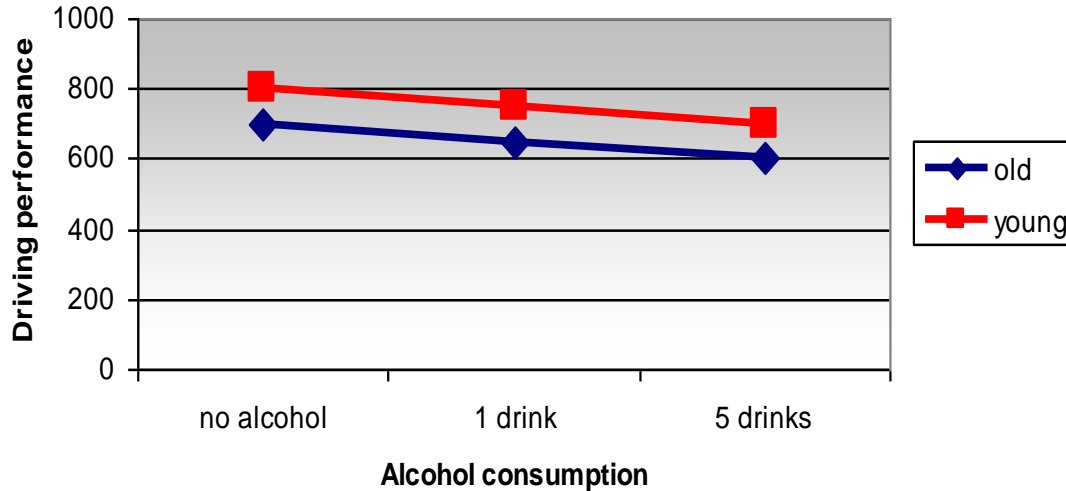
Women

No alc 1 drink 5 drinks

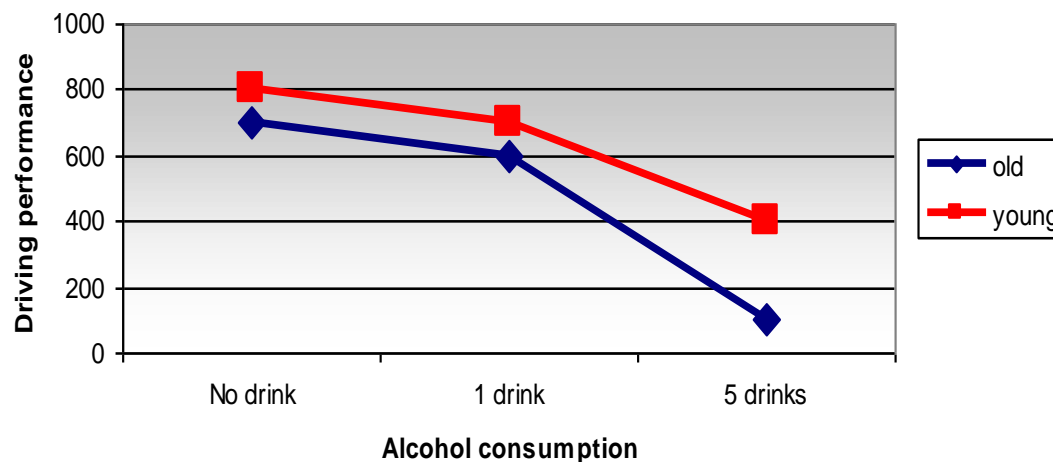
Old


Young


### Males



### Females



Graphical interp for 3-way:

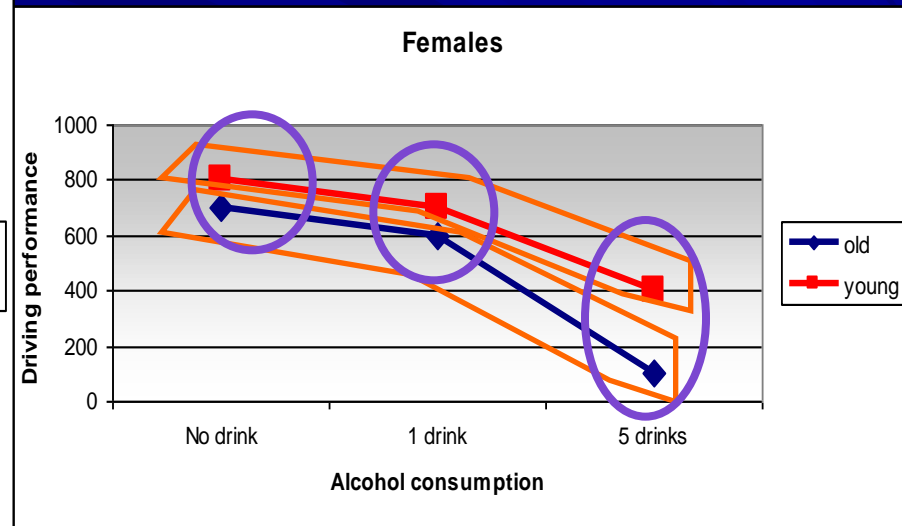
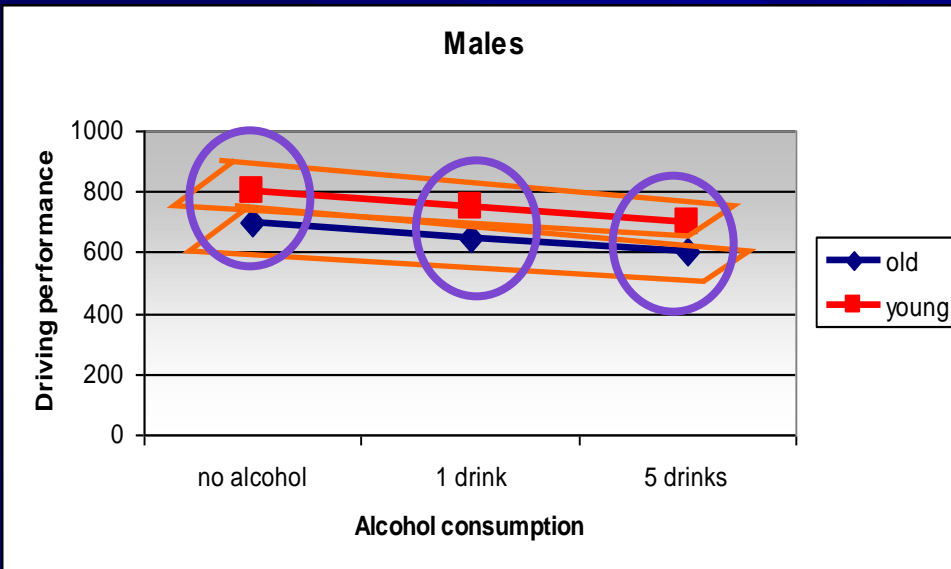
1. Plot 2-ways for each level of the third factor.

2. Check if pattern for one graph (simple interaction of AB at C1) is different from second graph (simple interaction of AB at C2). If graphs are not same pattern there is a 3-way interaction.

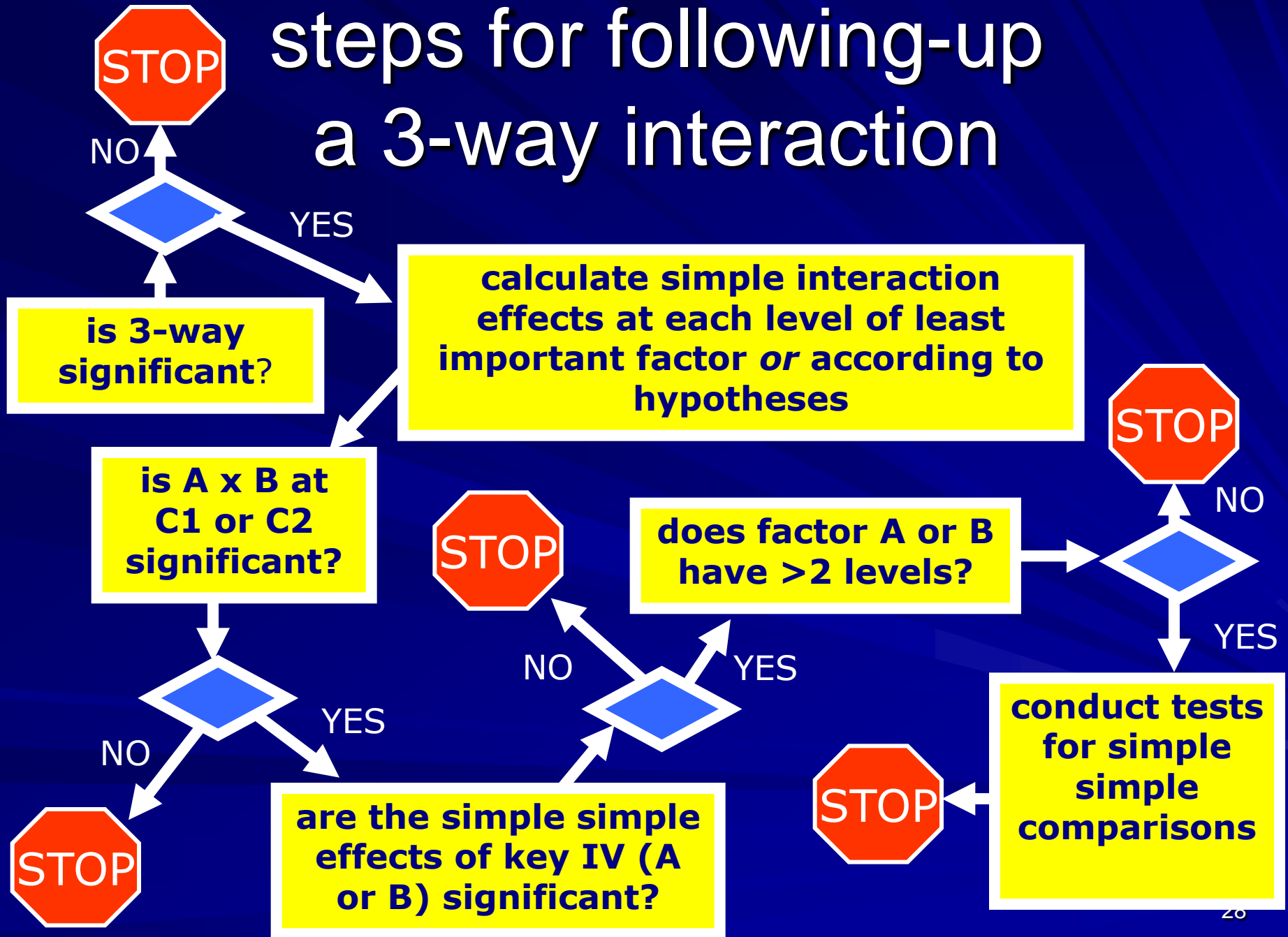
3. Difficult to interpret BC interactions or AC interactions from these, let alone MEs. Rely on statistical tests for lower-order effects.

## Sig 3-way interactions:

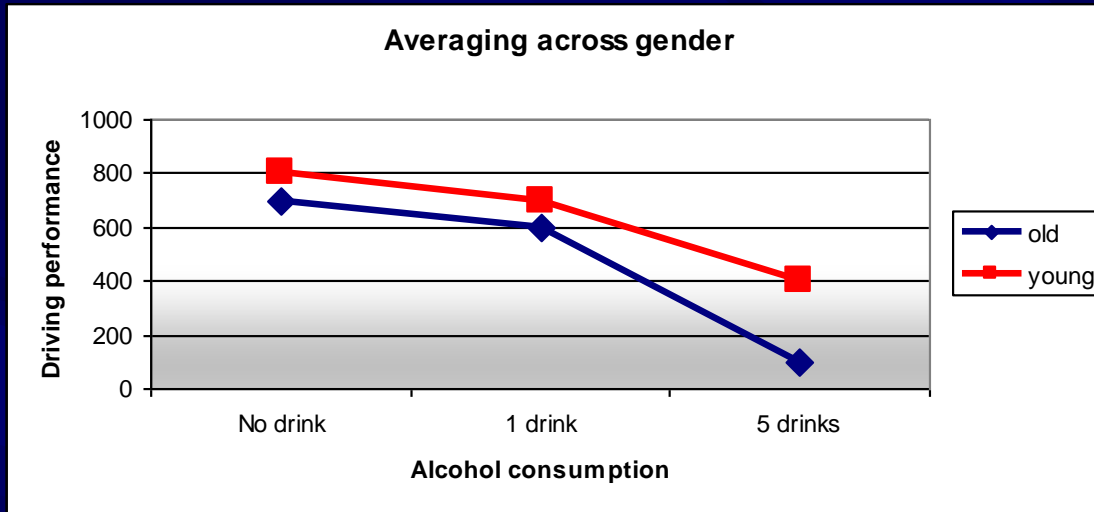
1. Mean that the effect of one IV changes depending on level of second variable, and how much depends on level of third variable (!).
2. Followed up with simple interactions (testing if effect of one IV changes depending on second, for each level of third IV).
  - NB, theory drives which simple interactions you follow up
3. Each sig simple interaction is followed up with simple simple effect tests (effect of IV at each level of 2<sup>nd</sup> variable at each level of 3<sup>rd</sup> – i.e., separately for each combo)
  - Theory drives which simple simple effects you test



# steps for following-up a 3-way interaction



Compare:



Vs. Sig 2-way interactions (averaging across 3<sup>rd</sup> factor):

1. Shows overall (averaging across 3<sup>rd</sup> variable), effect of one IV changes depending on level of second variable (lines not parallel).
2. Sig 2-way interaction still followed up with simple effect tests.



# example

- 3 Main effects (age, alcohol, sex)
- 3 two-way interactions (age\*alcohol, age\*sex, alcohol\*sex)
- 1 three-way interaction (age\*alcohol\*sex)

	Men			Women		
	No alc	1 drink	5 drinks	No alc	1 drink	5 drinks
Old						
Young						

# How many tests?

- 1 omnibus test in 1-way ANOVA
  - Test of IV
- 3 omnibus tests in 2-way ANOVA
  - Main effects of 2 IVs plus interaction
- 7 omnibus tests in 3-way ANOVA
  - 3 main effects, 3 two-ways, 1 three-way
- 15 omnibus tests in 4-way ANOVA
  - 4 MEs, 6 two-ways, 4 three-ways, 1 four-way
- Whoa!



# time for a new (quasi)experiment

## A test of the “Reinforcement Sensitivity Theory” of personality:

- some researchers have suggested that our personality is related to our capacity to learn from reward and punishment.
- people with an impulsive personality learn well from reward but not punishment, and people with an anxious personality learn well from punishment but not reward.
- possible gender differences are not clearly understood
- we construct a basic point-scoring reaction-time task measuring reactions time (RT) to investigate this theory
  - reward for fast responses or punishment for slow responses, plus a control condition where no reward/punishment is given
  - ½ of the participants have an anxious personality, ½ have an impulsive personality
  - ½ are male, ½ are female

# time for a new (quasi)experiment

there are a number of effects which might emerge:

- **main effects:**

- reinforcement (reward, punishment, none)
- personality (impulsive, anxious)
- gender (male, female)

- **two-way interactions** (also called first-order interactions):

- reinforcement x personality
- reinforcement x gender
- personality x gender

- **three-way interaction** (also called second-order interaction):

- reinforcement x personality x gender

# meanings of effects in 3-way designs

- **main effects:**

- differences between marginal means of one factor (averaging over levels of other factors)

- **two-way interactions:**

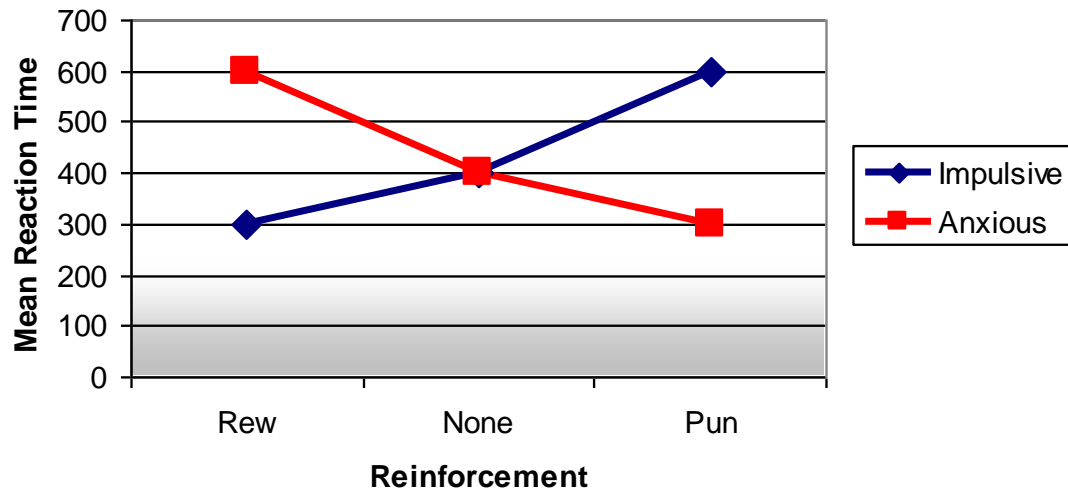
- examines whether the effect of one factor is the same at every level of another factor (averaging over levels of a third factor)

- ***three-way interaction:***

- examines whether the two-way interaction between two factors is the same at every level of the third factor

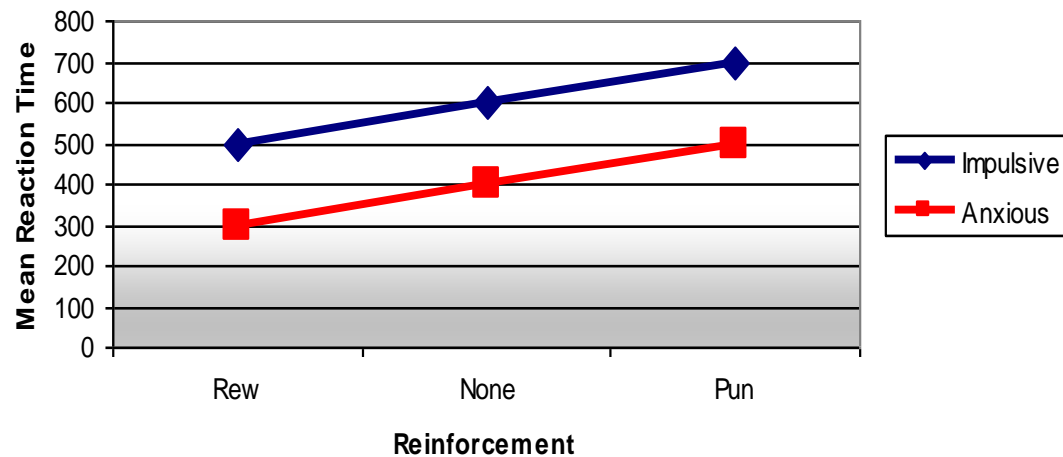


### Males



**here's how the  
results *might*  
look for the 3-  
way  
(Or....)**

### Females





# Note on hand calculations for the three-way

- You will not be assessed on them
  - For the rest of your career you will generally use SPSS or another stats package (tho' sometimes you can end up doing follow-ups / simple effects / simple comparisons by hand)
- Formulae plus example of hand calculations for three-way are posted in the resources section of the web site for you to look at
- However, you do need to know & understand the degrees of freedom for each effect (which means are being compared)

# data and cell totals/means (full layout)

Males			
Personality	Reinforcement		
	Rew	None	Pun
Impulsive	310	355	490
	320	350	495
	330	360	485
<b>Total</b>	<b>960</b>	<b>1065</b>	<b>1470</b>
<b>Mean</b>	<b>320</b>	<b>355</b>	<b>490</b>
Anxious	485	450	310
	490	455	320
	495	445	330
<b>Total</b>	<b>1470</b>	<b>1350</b>	<b>960</b>
<b>Mean</b>	<b>490</b>	<b>450</b>	<b>320</b>

Females			
Personality	Reinforcement		
	Rew	None	Pun
Impulsive	310	450	490
	320	455	486
	330	445	480
<b>Total</b>	<b>960</b>	<b>1350</b>	<b>1456</b>
<b>Mean</b>	<b>320</b>	<b>450</b>	<b>485</b>
Anxious	485	345	310
	480	350	320
	490	355	330
<b>Total</b>	<b>1455</b>	<b>1050</b>	<b>960</b>
<b>Mean</b>	<b>485</b>	<b>350</b>	<b>320</b>

# degrees of freedom

$$df_{\text{total}} = N - 1 = 36 - 1 = 35$$

$$df_P = p - 1 = 2 - 1 = 1$$

$$df_G = g - 1 = 2 - 1 = 1$$

$$df_R = r - 1 = 3 - 1 = 2$$

$$df_{PG} = (p - 1)(g - 1) = 1 \times 1 = 1$$

$$df_{RG} = (r - 1)(g - 1) = 2 \times 1 = 2$$

$$df_{PR} = (p - 1)(r - 1) = 1 \times 2 = 2$$

$$df_{PRG} = (p - 1)(g - 1)(r - 1) = 1 \times 1 \times 2 = 2$$

$$df_{\text{error}} = N - prg = 36 - 2 \times 3 \times 2 = 36 - 12 = 24$$

Regardless of # of factors in between-groups design, df for a factor always = # of levels - 1

Df for an interaction always multiply df for factors involved

↳ Df for error always  $N - \text{\#cells}$  or  $(n - 1) \times (\text{\# cells})$

# summary table (from SPSS)

## Tests of Between-Subjects Effects

Dependent Variable: RT

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Reinforcement	51.722	2	25.861	.517	.603
Personality	7.111	1	7.111	.142	.709
Gender	53.778	1	53.778	1.075	.310
Reinforcement x Personality	168516.722	2	84258.361	1684.232	.000
Reinforcement x Gender	.056	2	.028	.001	.999
Personality x Gender	9538.778	1	9538.778	190.670	.000
Reinforcement x Personality x Gender	19015.056	2	9507.528	190.045	.000
Error	1200.667	24	50.028		
Total	198383.889	35			

# no significant main effects

## Tests of Between-Subjects Effects

Dependent Variable: RT

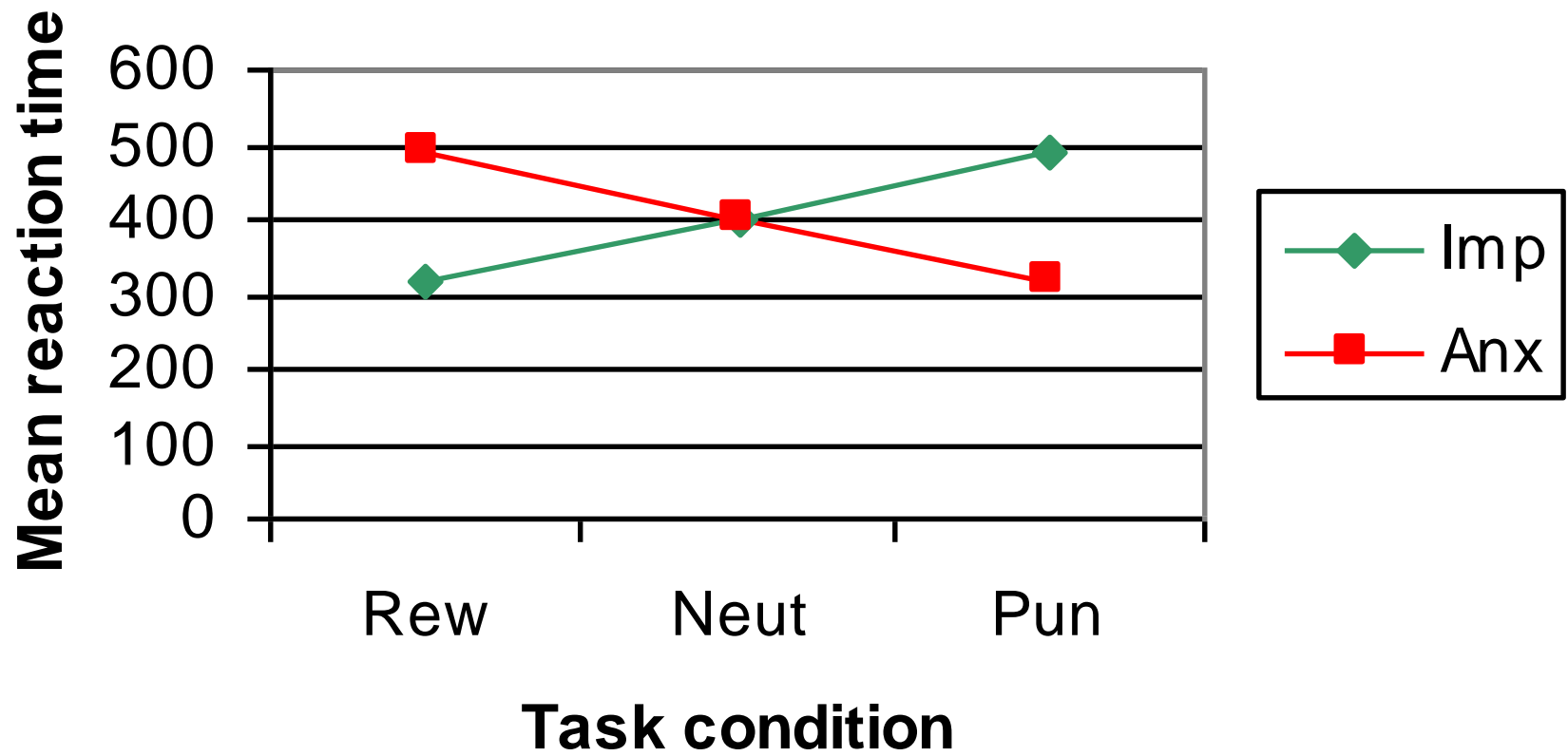
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<b>Personality</b>	<b>7.111</b>	<b>1</b>	<b>7.111</b>	<b>.142</b>	<b>.709</b>
<b>Gender</b>	<b>53.778</b>	<b>1</b>	<b>53.778</b>	<b>1.075</b>	<b>.310</b>
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# a significant 2-way interaction between personality and reinforcement

Tests of Between-Subjects Effects

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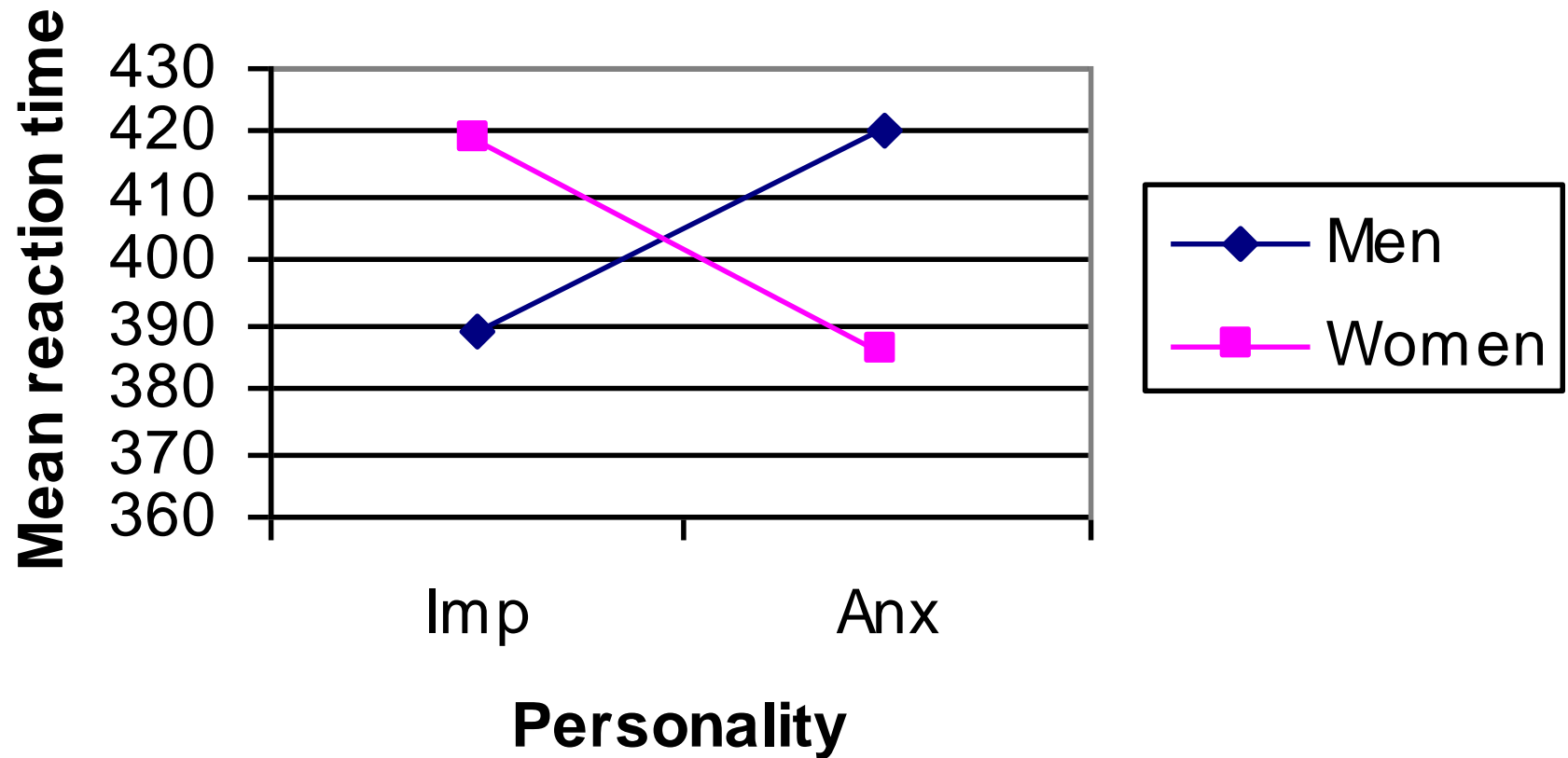


# a significant 2-way interaction between personality and gender

## Tests of Between-Subjects Effects

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Personality	7.111	1	7.111	.142	.709
Gender	53.778	1	53.778	1.075	.310
Reinforcement x Personality	168516.722	2	84258.361	1684.232	.000
Reinforcement x Gender	.056	2	.028	.001	.999
<b>Personality x Gender</b>	<b>9538.778</b>	<b>1</b>	<b>9538.778</b>	<b>190.670</b>	<b>.000</b>
Reinforcement x Personality x Gender	19015.056	2	9507.528	190.045	.000
Error	1200.667	24	50.028		
Total	198383.889	35			



# a significant 3-way interaction between reinforcement, personality and gender

Tests of Between-Subjects Effects

Dependent Variable: RT

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Reinforcement	51.722	2	25.861	.517	.603
Personality	7.111	1	7.111	.142	.709
Gender	53.778	1	53.778	1.075	.310
Reinforcement x Personality	168516.722	2	84258.361	1684.232	.000
Reinforcement x Gender	.056	2	.028	.001	.999
Personality x Gender	9538.778	1	9538.778	190.670	.000
<b>Reinforcement x Personality x Gender</b>	<b>19015.056</b>	<b>2</b>	<b>9507.528</b>	<b>190.045</b>	<b>.000</b>
Error	1200.667	24	50.028		
Total	198383.889	35			

# so there's this 3-way interaction...

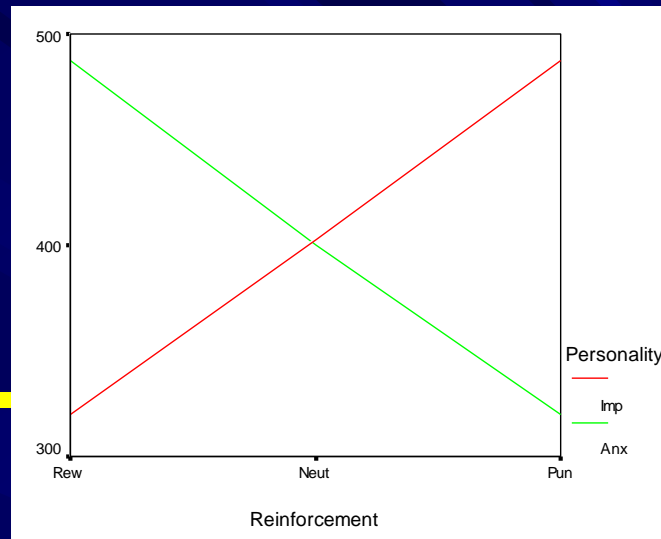
→ *what does that mean??*

- could be one of the following:
  - **personality X gender** 2-way interaction is different across levels of reinforcement
  - **reinforcement X personality** 2-way interaction is different across levels of gender
  - **reinforcement X gender** 2-way interaction is different across levels of personality

→ *need to focus your investigation*

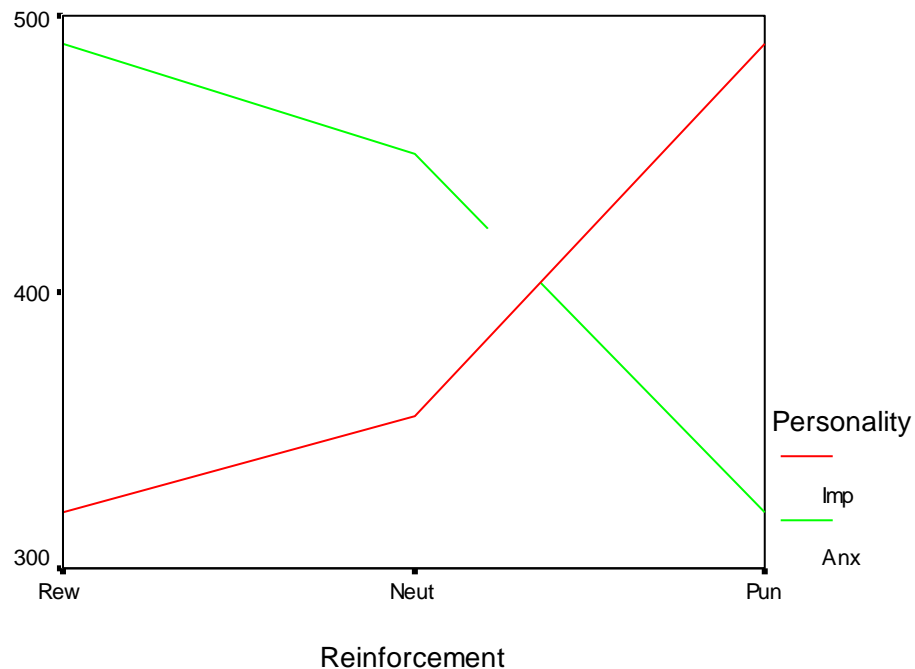
- 1) go back to theory and hypotheses
- 2) conduct follow-up analyses to test predictions

overall 2-way for  
Personality x  
Reinforcement ...

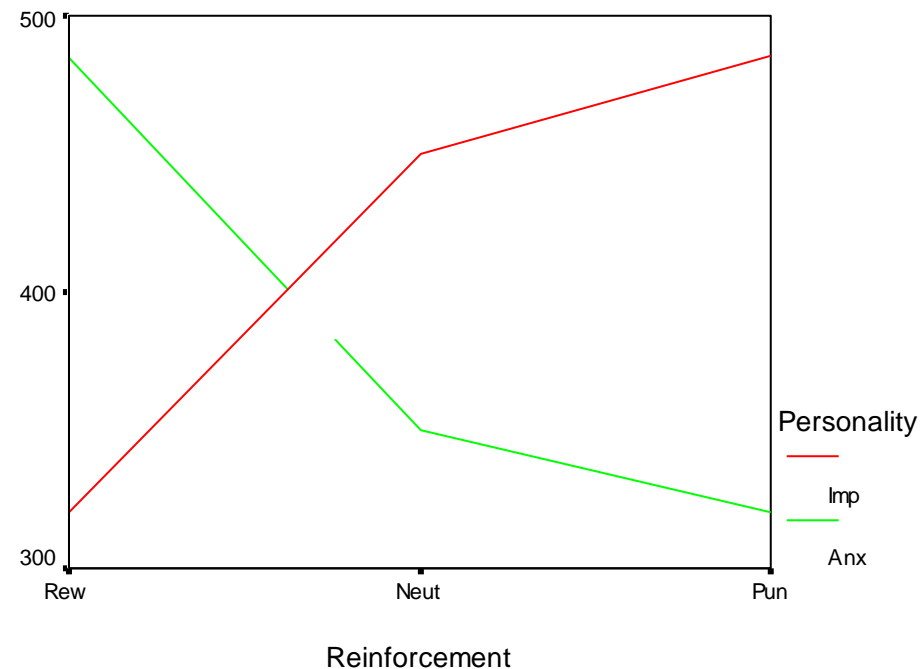


...is different  
at each level  
of gender

## Males



## Females



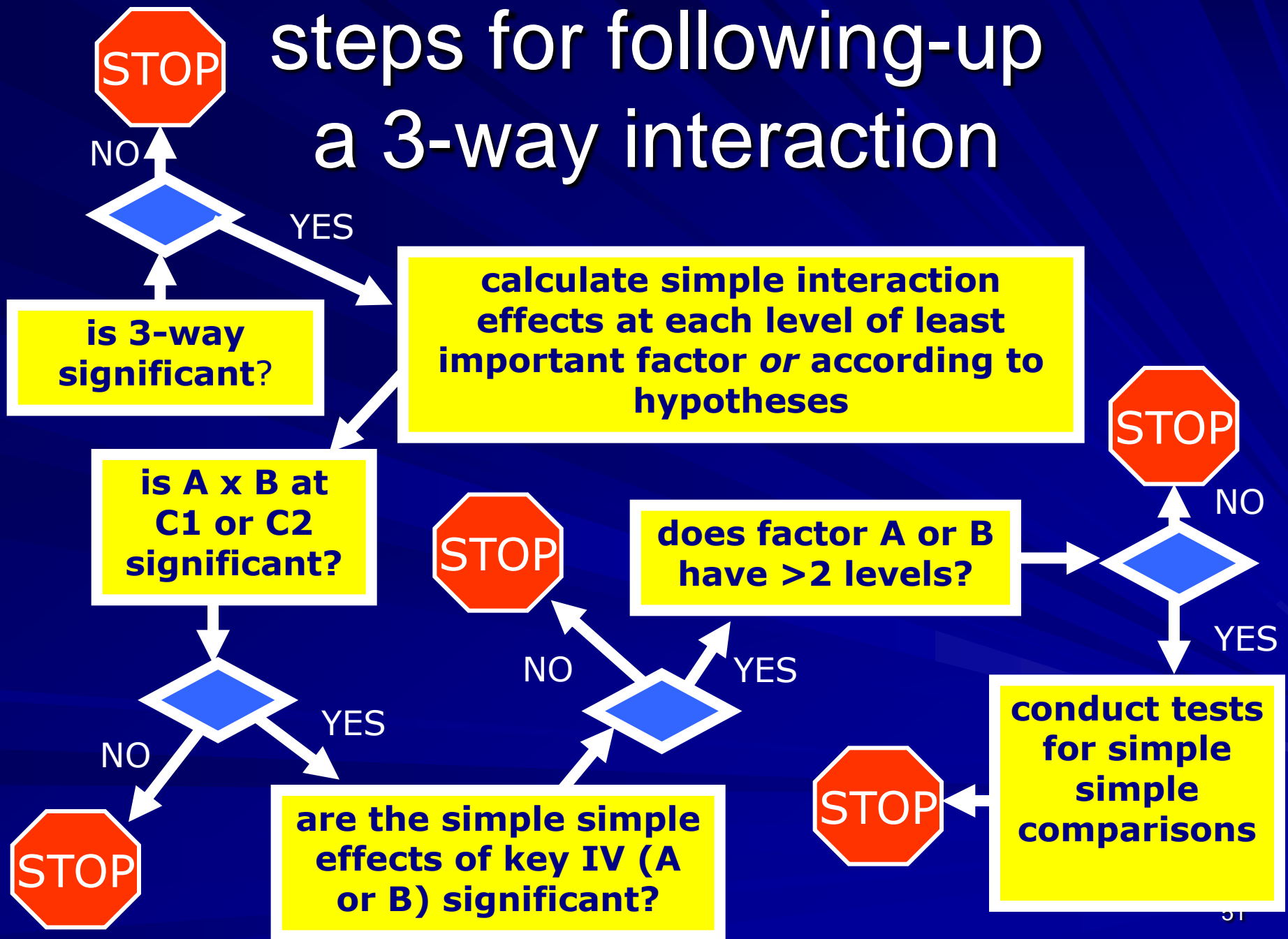


# following-up a 3-way anova

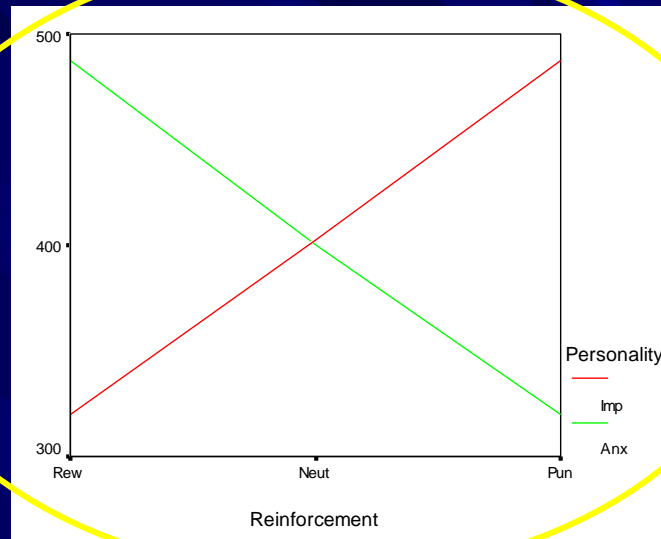
- **main effects with > 2 levels**
  - main effect comparisons - t-tests or linear contrasts
  - as per 2<sup>nd</sup> year stats and lecture 3
- **2-way interactions**
  - simple effects (as per lecture 3)
  - then, if simple effects are significant with > 2 levels, follow up with simple comparisons
- **3-way interactions**
  - *simple interaction effects* (new!)
  - if simple interaction effects are significant, follow up with simple simple effects (new!)
  - If simple simple effects are significant, follow up with simple simple comparisons (new!)



# steps for following-up a 3-way interaction

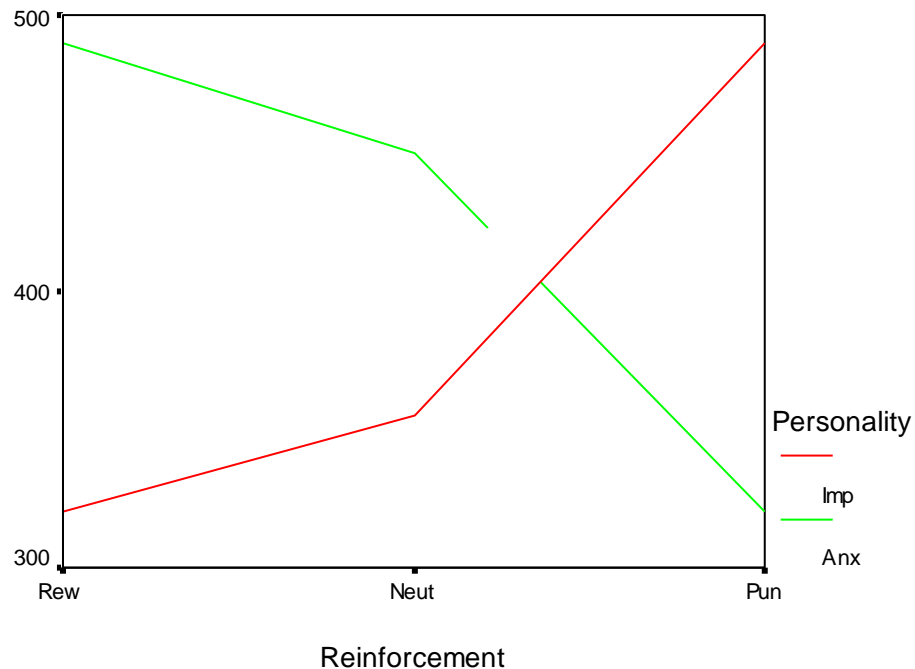


overall 2-way  
Vs simple 2-  
way  
interactions...

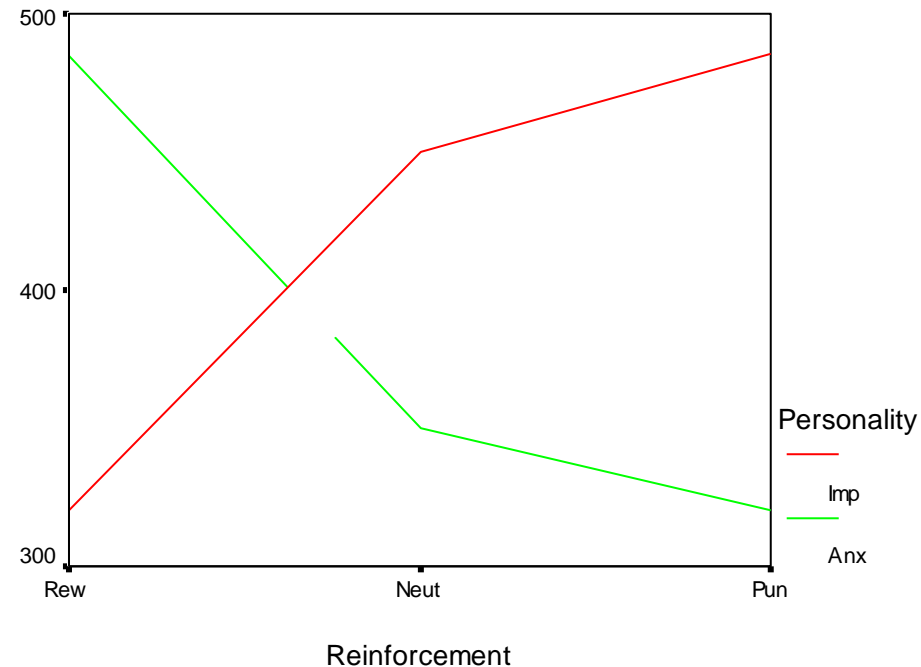


Test 2 way  
interaction with  
data averaged  
across gender

## Males



## Females

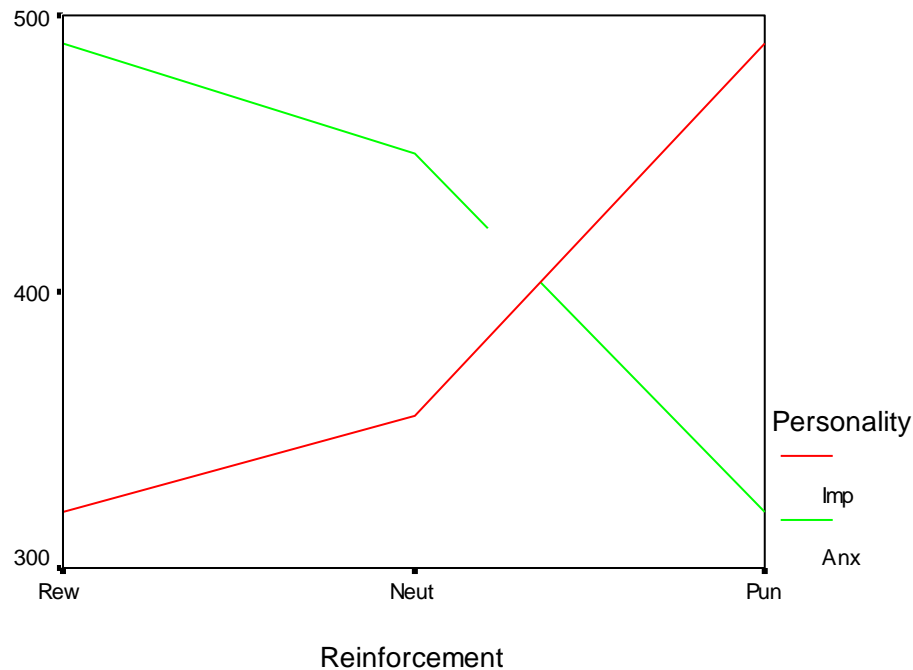


# simple interaction effects

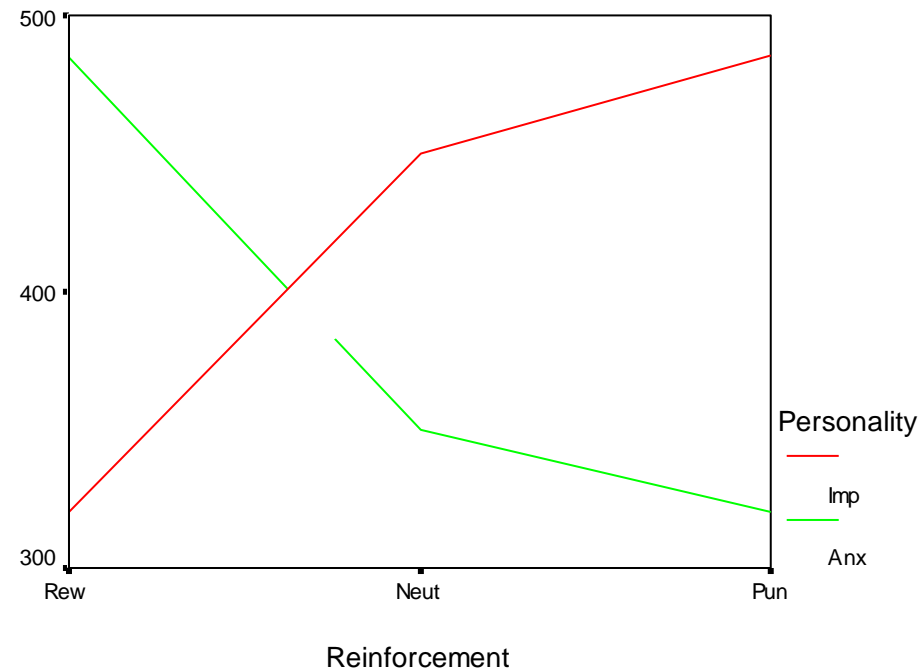
just as simple (*main*) effects are almost exactly the same as examining the 1-way treatment effect on factor A at each level of factor B, *simple interaction effects* are almost exactly the same as examining the 2-way interaction between factor A and B, at *each level of factor C*.

What distinguishes simple (main) effects from multiple 1-way anova treatment effects and simple interaction effects from 2-way interactions is that simple main and interaction effects use  $MS_{\text{error}}$  from the overall anova as the error term

## Males



## Females



the graphs depicting the 2x2x3 interaction between gender, personality and reinforcement also provide a visual representation of the simple interaction effects we would conduct – a simple personality x reinforcement interaction at the two levels of gender

Males			
Personality	Reinforcement		
	Rew	None	Pun
Impulsive	310	355	490
	320	350	495
	330	360	485
<b>Total</b>	<b>960</b>	<b>1065</b>	<b>1470</b>
<b>Mean</b>	<b>320</b>	<b>355</b>	<b>490</b>
Anxious	485	450	310
	490	455	320
	495	445	330
<b>Total</b>	<b>1470</b>	<b>1350</b>	<b>960</b>
<b>Mean</b>	<b>490</b>	<b>450</b>	<b>320</b>

Females			
Personality	Reinforcement		
	Rew	None	Pun
Impulsive	310	450	490
	320	455	486
	330	445	480
<b>Total</b>	<b>960</b>	<b>1350</b>	<b>1456</b>
<b>Mean</b>	<b>320</b>	<b>450</b>	<b>485</b>
Anxious	485	345	310
	480	350	320
	490	355	330
<b>Total</b>	<b>1455</b>	<b>1050</b>	<b>960</b>
<b>Mean</b>	<b>485</b>	<b>350</b>	<b>320</b>

so does the original data table– this is just what we would have if we ran 2 separate 2-way anovas

so...in the case of examining the two way interaction between Personality and Reinforcement FOR MALES, it is just as if we had no females in the study:

Males				
Personality	Reinforcement			Marginal
	Rew	None	Pun	
Impulsive	310	355	490	
	320	350	495	
	330	360	485	
<b>Total</b>	<b>960</b>	<b>1065</b>	<b>1470</b>	<b>3495</b>
<b>Mean</b>	<b>320</b>	<b>355</b>	<b>490</b>	
Anxious	485	450	310	
	490	455	320	
	495	445	330	
<b>Total</b>	<b>1470</b>	<b>1350</b>	<b>960</b>	<b>3780</b>
<b>Mean</b>	<b>490</b>	<b>450</b>	<b>320</b>	
Marginal Totals	<b>2430</b>	<b>2415</b>	<b>2430</b>	<b>7275</b>

# simple interaction effects

But  $F$  tests for simple interaction effects are **not** the same as  $F$  tests for 2-way interactions

simple personality X reinforcement 2-way interactions:

- $MS_{\text{error}}$  taken from 3-way omnibus ANOVA table

overall personality X reinforcement 2-way interactions:

- $MS_{\text{error}}$  separate value for men and women (taken from each 2-way omnibus ANOVA table)



# summary table for simple interaction effects

Source	SS	df	MS	F	p
PR at G1	95725.00	2	47862.50	956.72	0.000
PR at G2	91806.78	2	45903.39	917.56	0.000
Error	1200.67	24	50.03		

critical F at  $\alpha=.05$  (2,24) = 3.40

Same error & df as original ANOVA

Calculated by hand (see  
web resources) or via  
SPSS

Degrees of freedom for a simple interaction effect are just the df for the associated interaction

$$df = df_{PR} (2-1)(3-1) = 2$$

These are your calculated SS values

Source	SS	df	MS	F	p
PR at G1	95725.00	2	47862.50	956.72	0.000
PR at G2	91806.78	2	45903.39	917.56	0.000
Error	1200.67	24	50.03		

critical F at  $\alpha=.05$  (2,24) = 3.40

$SS_{error}$  term (and df) is taken from the main 2 x 2 x 3 anova

Mean Squares and F values calculated as per usual

Indicates that the personality x reinforcement interaction is significant for males and females. Each significant simple interaction needs following up with **simple simple effect tests...**



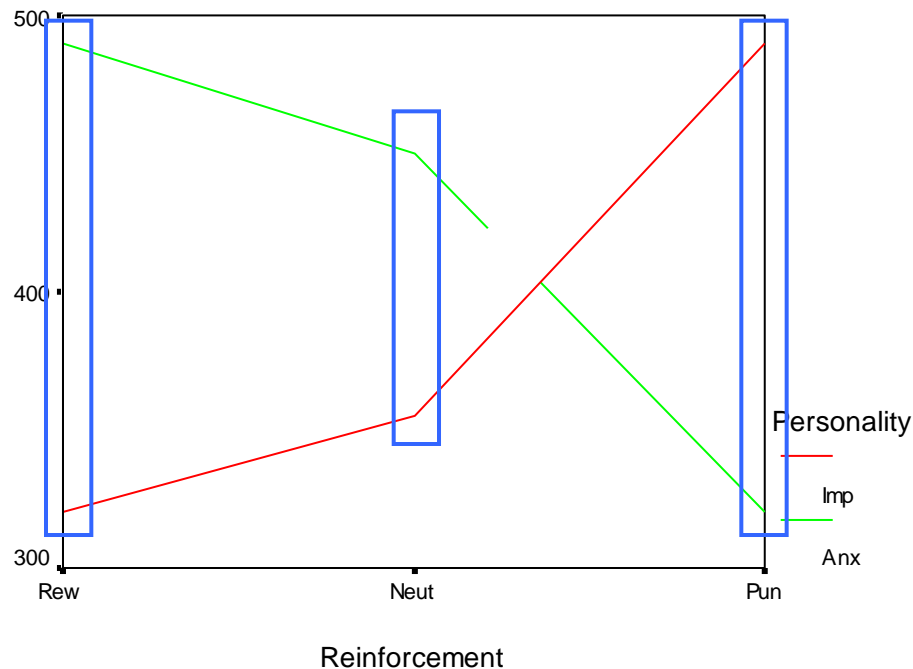
# simple simple effects

remember, simple effects examine the effect of factor A at each level of factor B

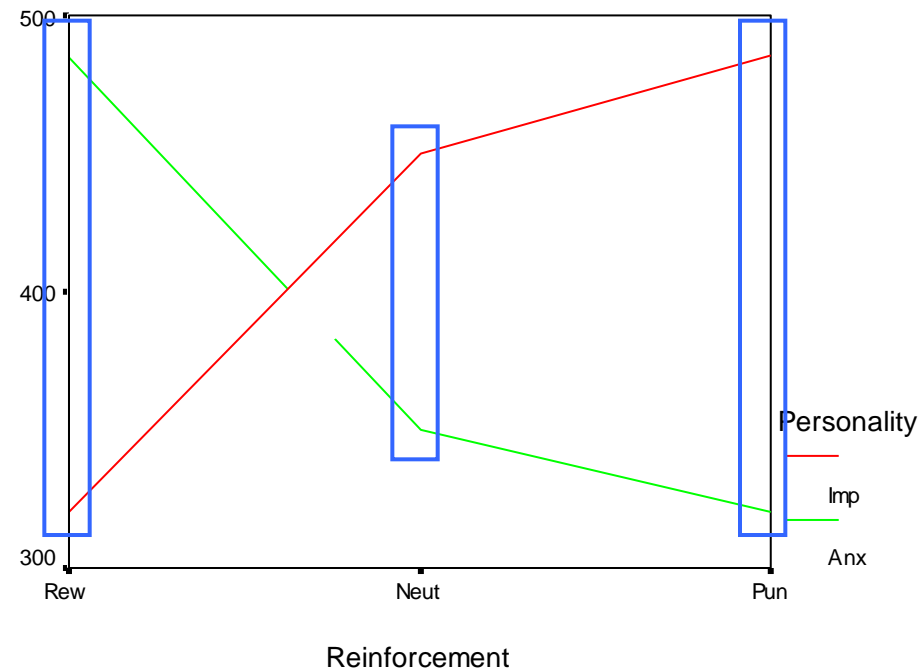
*simple simple effects* are *exactly* the same as ordinary simple effects except the effect of factor A at each level of factor B, is examined *at each level of factor C*.

again,  $MS_{\text{error}}$  from the overall anova is the error term

## Males



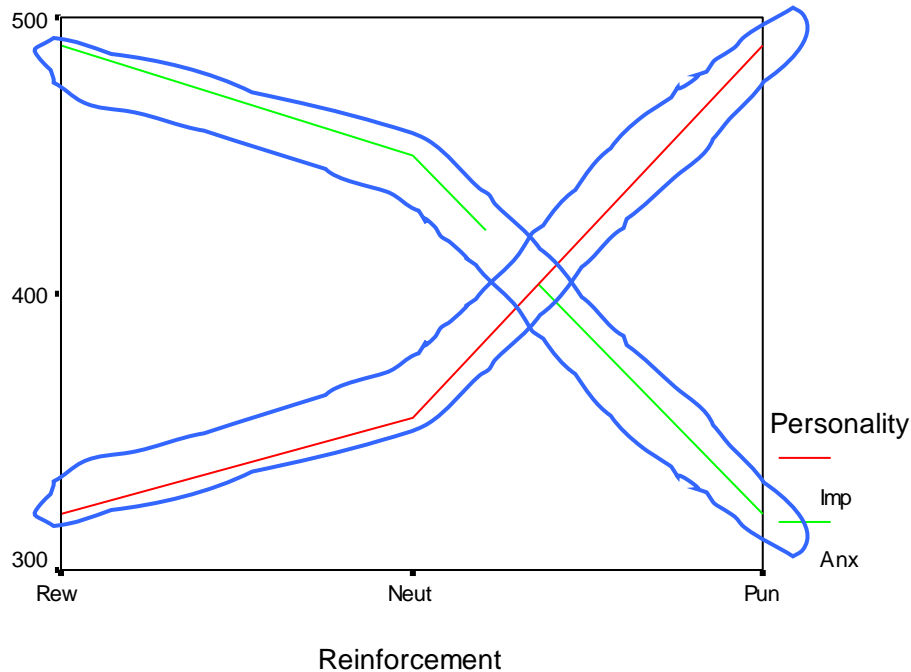
## Females



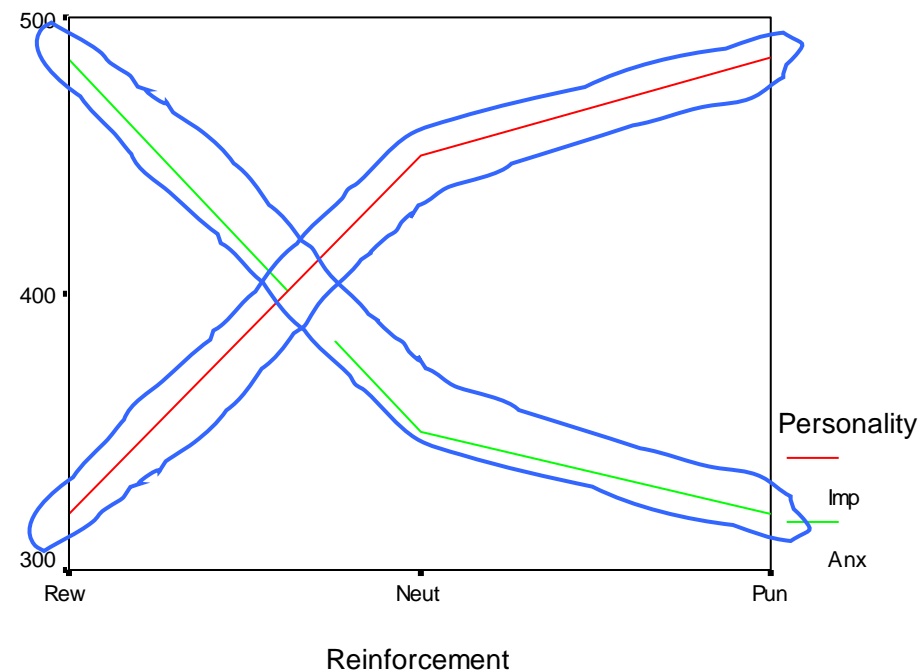
hence, for both males and females, the effect of personality at each level of reinforcement was significant (although *opposite* under neutral reinforcement!)

could also then compute the simple simple effects of **reinforcement** at each level of personality (for males and females)...

## Males



## Females



hence, for both males and females, the effect of reinforcement at each level of personality was significant

could then follow up the simple simple effect of reinforcement with **simple simple comparisons** to see which levels of reinforcement differ within each level of personality (for males and females)....

# simple simple comparisons

*exactly* the same as ordinary simple comparisons / contrasts except we compute for each level of a third factor.

the same formula from Lecture 3 can be used:

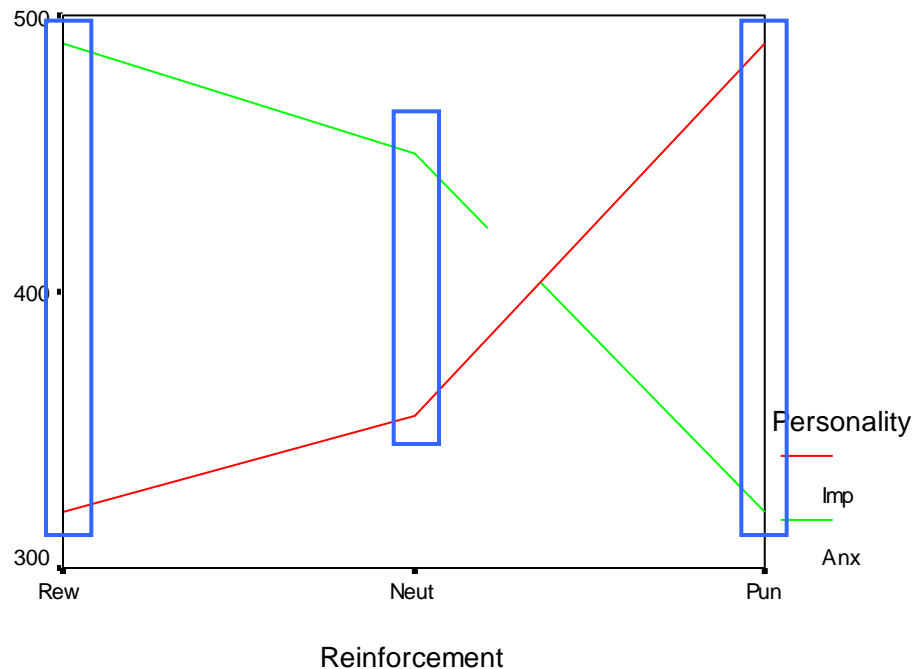
$$t = \frac{L}{\sqrt{\frac{\sum a_j^2 MS_{error}}{n}}}$$

$$L = \sum a_j \bar{X}_j$$

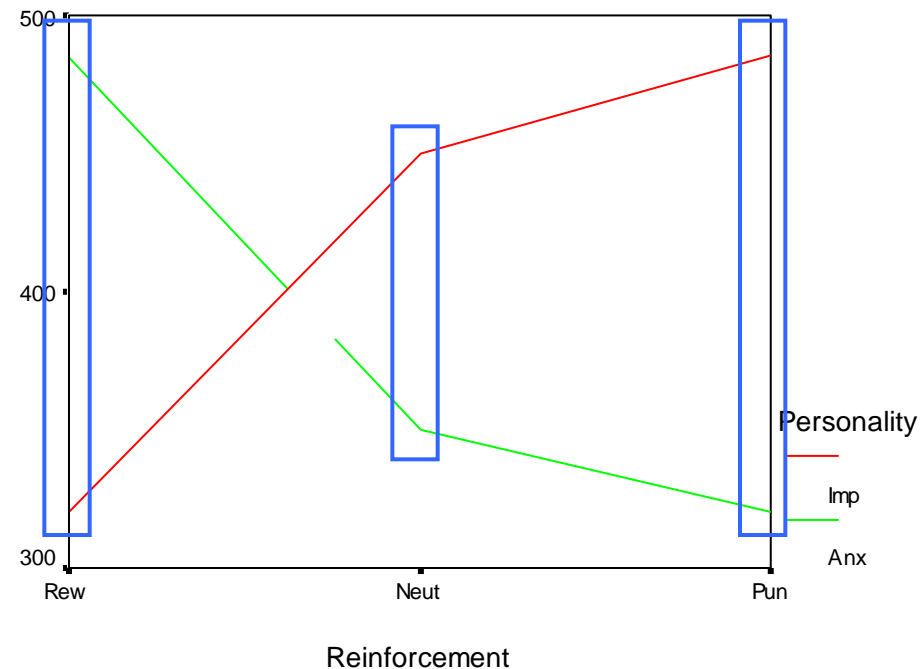
$$df_{error} = N - ab$$



## Males



## Females



hence, for both males and females, the effect of personality at each level of reinforcement was significant (although *opposite* under neutral reinforcement!)

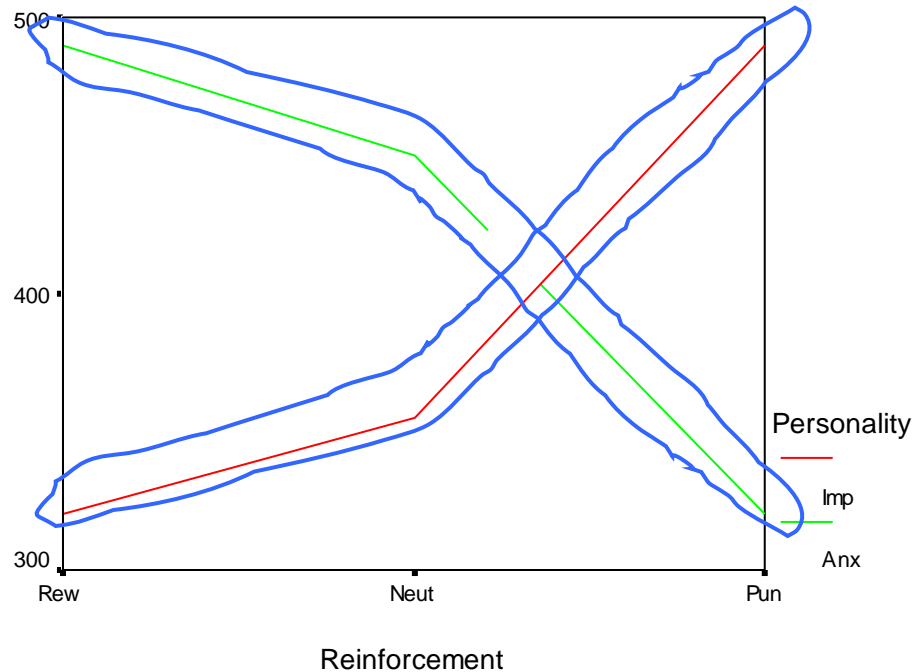
could also then compute the simple simple effects of **reinforcement** at each level of personality (for males and females)...

# summary table for simple simple effects of personality, at each level of reinforcement, for males and females

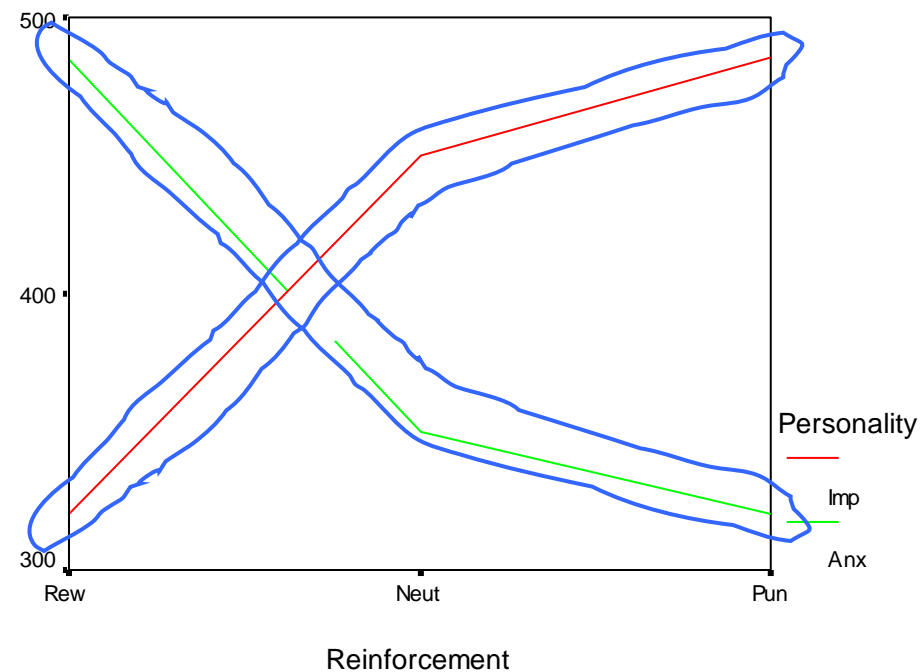
<b>Source</b>	<b>SS</b>	<b>df</b>	<b>MS</b>	<b>F</b>	<b>p</b>
<b>P at R1 at G1</b>	43350.00	1	43350.00	866.52	0.000
<b>P at R2 at G1</b>	13537.50	1	13537.50	270.60	0.000
<b>P at R3 at G1</b>	4330.50	1	4330.50	86.56	0.000
<b>P at R1 at G2</b>	40837.50	1	40837.50	816.30	0.000
<b>P at R2 at G2</b>	15000.00	1	15000.00	299.83	0.000
<b>P at R3 at G2</b>	41002.7	1	41002.66	819.60	0.000
<b>Error</b>	1200.67	24	50.03		

critical F at  $\alpha=.05$  (1,24) = 4.26

## Males



## Females



hence, for both males and females, the effect of reinforcement at each level of personality was significant

could then follow up the simple simple effect of reinforcement with **simple simple comparisons** to see which levels of reinforcement differ within each level of personality (for males and females)....

# summary table

simple simple effects of reinforcement, at each level of personality, for males and females

Source	SS	df	MS	F	p
R at P1 at G1	48350.00	2	24175.00	483.23	0.000
R at P2 at G1	47400.00	2	23700.00	473.74	0.000
R at P1 at G2	45483.56	2	22741.78	454.58	0.000
R at P2 at G2	46350.00	2	23175.00	463.24	0.000
Error	1200.67	24	50.03		

critical F at  $\alpha=.05$  (2,24) = 3.40

# simple simple comparisons

*exactly* the same as ordinary simple comparisons / contrasts except we compute for each level of a third factor.

the same formula from Lecture 3 can be used:

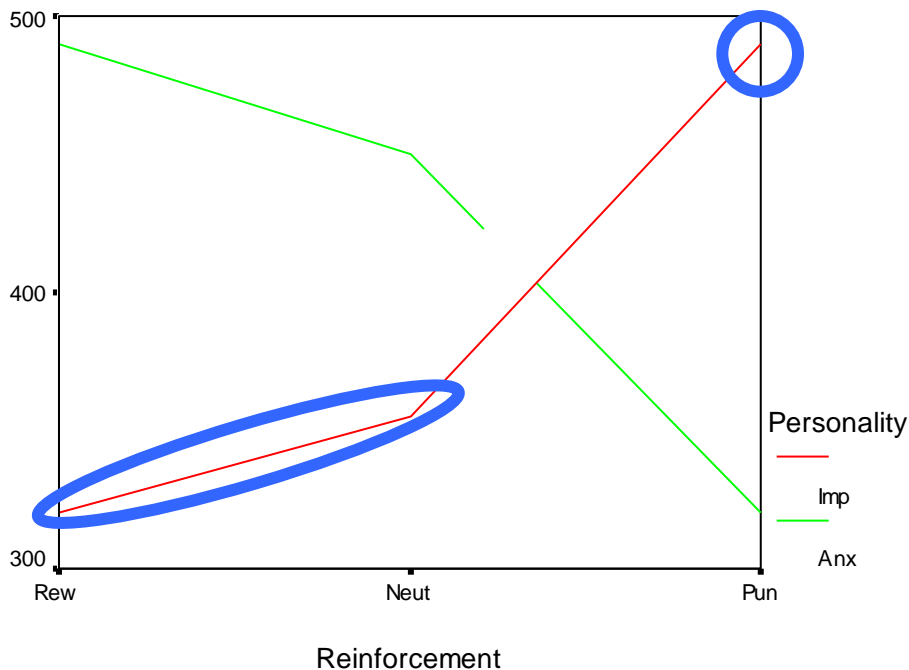
$$t = \frac{L}{\sqrt{\frac{\sum a_j^2 MS_{error}}{n}}}$$

$$L = \sum a_j \bar{X}_j$$

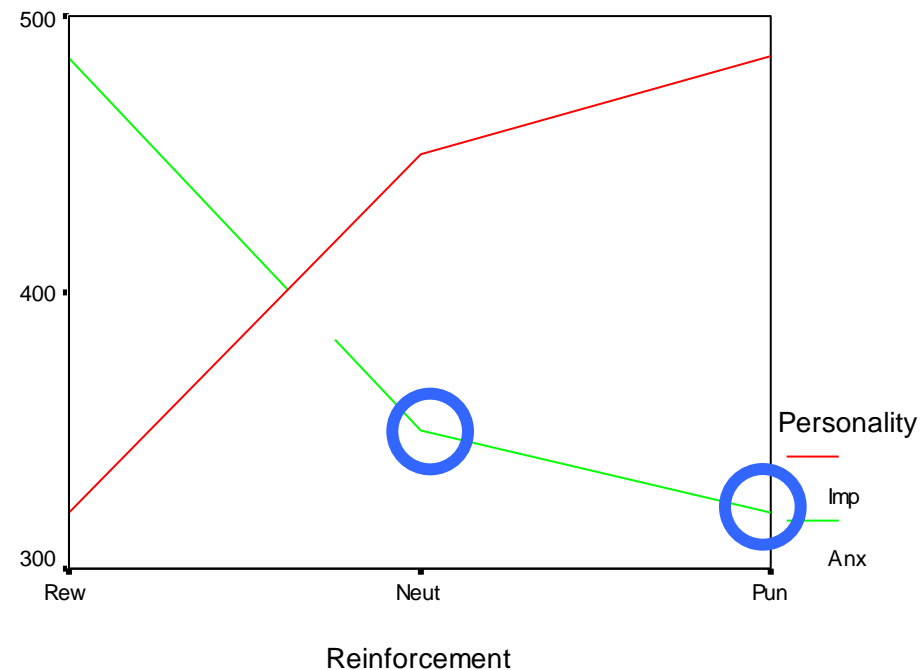
$$df_{error} = N - ab$$

# some possible comparisons...

## Males



## Females



$R_1$  and  $R_2$  vs  $R_3$  at  $P_1$  for  $G_1$  ...

$R_2$  vs  $R_3$  at  $P_2$  for  $G_1$  ...etc

# simple simple comparisons for *reinforcement* at each level of *personality* (for *males*)

<b>Males</b>	Consumption		
	Rew	None	Pun
<b>Impusivity</b>	320	355	490
Contrast 1	1	-1	0
Contrast 2	1	1	-2
<b>Anxiety</b>	490	450	320
Contrast 1	1	-1	0
Contrast 2	1	1	-2

**Note: these are slightly different contrasts to the ones from Lecture 3 – the exact comparisons you make will depend upon your theory**

# Calculations for impulsivity contrast 1

<b>Males</b>	Consumption		
	Rew	None	Pun
<b>Impusivity</b>	320	355	490
Contrast 1	1	-1	0
Contrast 2	1	1	-2
<b>Anxiety</b>	490	450	320
Contrast 1	1	-1	0
Contrast 2	1	1	-2

$$L = 1(320) - 1(355) + 0(490) = \mathbf{-35.00}$$

$$t = \frac{-35.00}{\sqrt{\frac{(+1^2 + (-1)^2 + 0^2)50.03}{3}}} = -6.06$$

$$t'_{\alpha=.05} (24) = 2.39$$

(with Bonferroni adjustment for 2 comparisons)



...and so on for

- impulsivity contrast 2...
- Anxiety contrast 1...
- Anxiety contrast 2...
- then all four contrasts  
for females...

SPC3010

# count the number of tests we've just conducted

- **omnibus tests**
  - 7 (3 main effects, 3 two-way interactions, 1 three-way interaction)
- **simple interaction effects**
  - 2 (personality x reinforcement at each level of gender)
- **simple simple effects**
  - 10 (6 for personality (at each level of reinforcement) for males and females, 4 for reinforcement (at each level of personality) for males and females)
- **simple simple comparisons**
  - 8 (2 comparisons for each personality condition for males and females)
- ***total = 27 tests!!!***
  - each with a *type-1 error rate* of .05!!!
  - this leads to a *familywise error rate* of  $27 * .05 = .7$ , or 135% (*lets just say 'high'!*)

# take-home message

- **conducting an exhaustive set of follow-up tests for higher-order factorial designs can inflate familywise alpha (and is very tedious!)**
- **ultimately, there is no simple rule:** what you report depends **entirely** upon your research predictions
  - in our case we had (implicitly) predicted the Personality x Reinforcement interaction, and we were going to see if this interaction was the same for males and females
    - people with an impulsive personality learn well from reward but not punishment, and people with an anxious personality learn well from punishment but not reward.
    - Possible gender differences not well understood.
  - hence our write up might have gone something like this...

# reporting

I haven't put effect sizes. These would be required for all tests these days.

“The predicted interaction was significant,  $F(2, 24) = 1684.23, p < .001$ , but this was qualified by 3-way interaction among personality, reinforcement, and gender  $F(2, 24) = 190.67, p < .001$ . Simple interaction analyses revealed the personality x reinforcement interaction was significant for both males,  $F(2, 24) = 956.72, p < .001$ , and females,  $F(2, 24) = 917.56, p < .001$ . The simple simple effects of personality were then analysed for each level of gender and reinforcement, and Table 1 presents the relevant means. For both genders, as predicted, under punishment anxious participants were faster than impulsive participants,  $F_s > 819.58, p_s < .001$ , while under reward impulsive participants were faster than anxious participants,  $F_s > 816.29, p_s < .001$ . However, in the neutral reinforcement condition the gender difference emerged: impulsive males performed *better* than anxious males  $F(1, 24) = 270.60, p < .001$  ( $M_s = 355, 450$ ), while impulsive females performed *worse* than anxious females,  $F(2, 24) = 299.83, p < .001$  ( $M_s = 450, 350$ ).”

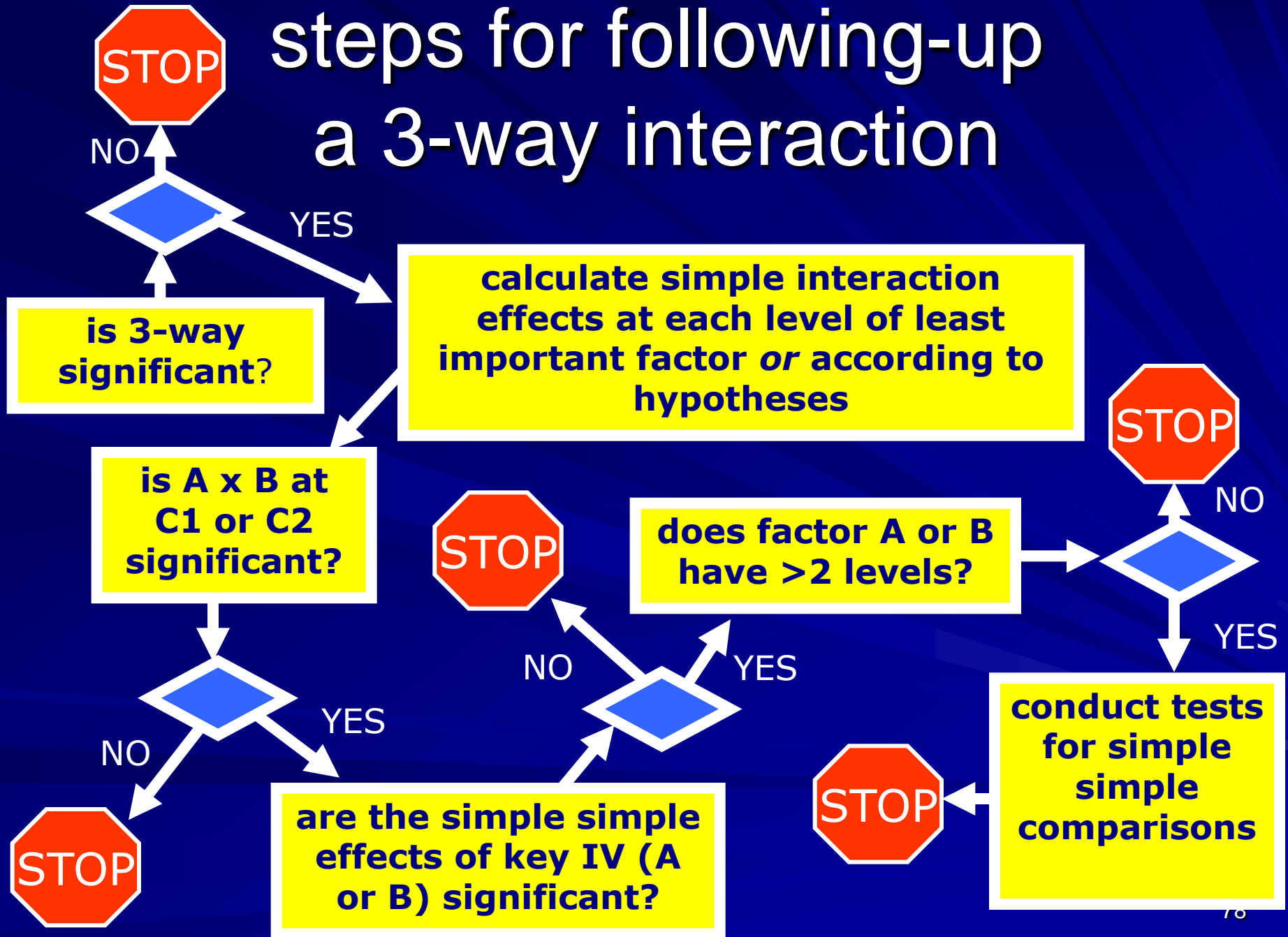


Table 1. Mean reaction time as a function of personality, reinforcement, and gender.

		Personality Type	
		Impulsive	Anxious
Reinforcement:			
Punishment			
	Women	485.33 <sub>a</sub>	320.00 <sub>a</sub>
	Men	490.00 <sub>a</sub>	320.00 <sub>a</sub>
None			
	Women	450.00 <sub>a</sub>	350.00 <sub>a</sub>
	Men	355.00 <sub>a</sub>	450.00 <sub>a</sub>
Reward			
	Women	320.00 <sub>a</sub>	485.00 <sub>a</sub>
	Men	320.00 <sub>a</sub>	490.00 <sub>a</sub>

Note. Subscripts within the row indicate significant simple effects of personality.

# steps for following-up a 3-way interaction



# summary

- 3-way interactions are very complex!
- this increasing complexity highlights the need for analyses to be driven by your hypotheses
- it also foreshadows the usefulness of computerised statistical packages like SPSS (which you will start using in tutes next week!)

### Next week:

- Power analysis

### Readings for this week:

- Howell chapter 13
  - especially section 13.12
- Field Chapter 10 (and look through SPSS stuff-i.e. sections 10.3-onwards for next week's tutorial!)
- Field Chapter 2 (a good introduction to SPSS for the tutes next week)

### In the tutes:

- This week: Hand calculations for follow-ups
- Next week – SPSS tute!